Name:

Student number:

1	
2	
3	
4	
5	
Σ	

METU MATH 111, Midterm 1

Thursday, November 3, 2011, at 17:40 (100 minutes), totally 60 points

Instructors: E.Emelyanov, G.Ercan, S.Finashin, E.Solak

Instructions: Please, show clearly the logic of your solutions.

Problem 1. (4+4+4 pts) (a) Is the statement $(P \leftrightarrow Q) \wedge (P \wedge \neg Q)$ a tautology, a contradiction, or neither? (Explain.)

(b) Show that statement $P \to (Q \vee R)$ is equivalent to $(P \wedge \neg Q) \to R$.

(c) Record (using the logical symbols) the following claim.
For every a, the equation $ax^2 + 4x - 2 = 0$ has at least one solution iff $a \geq -2$.

Problem 2. (3+3+3+3 pts) Suppose that $P(x, y)$ means "x divides y", and the universe is the set of positive integers. Determine whether the following statements are true or false. Explain (briefly).

(a) $\exists y \, \forall x \; P(x, y)$

(b) $\exists x \, \forall y \; P(x, y)$

(c) $\exists x \, \exists y \; P(x, y)$

(d) $\forall x \, \forall y \; P(x, y)$

Problem 3. (6+6 pts) Negate the following statements and simplify the result (the answer may contain negation only of P, Q, and R).

(a) $(P \vee \neg Q) \wedge \neg R$

(b) $\forall x \; (P(x) \rightarrow Q(x))$

Problem 4. (4+8 pts) Suppose that A, B, and C are sets.
(a) State that $A \cap B \cap C \neq \varnothing$ (use an appropriate quantifier and logical connectives).

(b) Prove that $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$. Express clearly the logic of your proof!

Problem 5. (6+6 pts) Prove the following propositions. (Express clearly your logic).
(a) If n is an integer and $n^3 + 5$ is odd, then n is even.

(b) There does not exist a positive real number a such that $a + \frac{1}{a} < 2$.