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# METU MATH 111, Midterm 1

Thursday, November 3, 2011, at 17:40 (100 minutes), totally 60 points

**Instructors:** E.Emelyanov, G.Ercan, S.Finashin, E.Solak

**Instructions:** Please, show clearly the logic of your solutions.

**Problem 1. (4+4+4 pts)** (a) Is the statement $(P \leftrightarrow Q) \wedge (P \wedge \neg Q)$ a tautology, a contradiction, or neither? (Explain.)

(b) Show that statement $P \to (Q \vee R)$ is equivalent to $(P \wedge \neg Q) \to R$.

(c) Record (using the logical symbols) the following claim.
For every $a$, the equation $ax^2 + 4x - 2 = 0$ has at least one solution iff $a \geq -2$.

**Problem 2. (3+3+3+3 pts)** Suppose that $P(x, y)$ means "$x$ divides $y$", and the universe is the set of positive integers. Determine whether the following statements are true or false. Explain (briefly).
(a) $\exists y\,\forall x\ P(x, y)$

(b) $\exists x\,\forall y\ P(x, y)$

(c) $\exists x\,\exists y\ P(x, y)$

(d) $\forall x\,\forall y\ P(x, y)$

**Problem 3. (6+6 pts)** Negate the following statements and simplify the result (the answer may contain negation only of $P$, $Q$, and $R$).
(a) $(P \vee \neg Q) \wedge \neg R$

(b) $\forall x\ (P(x) \to Q(x))$

**Problem 4. (8 pts)** Suppose that $A$, $B$, and $C$ are sets.
Prove that $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$. Express clearly the logic of your proof!

**Problem 5. (6+6 pts)** Prove the following propositions. (Express clearly your logic).
(a) If $n$ is an integer and $n^3 + 5$ is odd, then $n$ is even.

(b) There does not exist a positive real number $a$ such that $a + \frac{1}{a} < 2$.