M E T U
Department of Mathematics

Group	Fundamentals of Mathematics Midterm 2	List No.

Code : *Math 111* Acad. Year : *2013* Semester : *Fall* Instructor : *G.Ercan, S.Finashin, M.Kuzucuoğlu, Ö.Küçüksakallı.* Date : *December 19, 2013* Time : *17:40* Duration : *100 minutes*	Last Name : Name : Student No. : Department : Section : Signature : 6 QUESTIONS ON 4 PAGES 60 TOTAL POINTS

1	2	3	4	5	6		

1. (12pts) Let $f : \mathbb{Z} \to \mathbb{Z}$ be the function defined by $f(x) = 2x + 3$. Define a relation R on $\mathbb{Z}$ by xRy if and only if $f(x) \equiv f(y) (\text{mod } 5)$ for any x, y in $\mathbb{Z}$.

(a) Prove that R is an equivalence relation on $\mathbb{Z}$.

(b) Describe the R-equivalence class [0] explicitly.

2. (10pts) (a) Define the function $f : \mathbb{Z} \to \mathbb{Z}$ by $f(x) = 7x - 2$. Determine whether f is injective, surjective and bijective.

(b) Define the function $g : \mathbb{Q} \to \mathbb{Q}$ by $g(x) = 7x - 2$. Determine whether g is injective, surjective and bijective.

3. (10pts) Prove that a function $f : A \to B$ has a left inverse if and only if f is injective.

4. (6pts) Give an example of subsets A, B and C of $\mathbb{Z}$ such that $A - (B - C) \neq (A - B) - C$.

5. (10pts) Prove that if A, B and C are sets, then $A \times (B - C) = (A \times B) - (A \times C)$.

6. (12pts) Consider the poset $(\mathcal{P}(\mathbb{Z}), \subseteq)$ and let $A = \{\{4\}, \{1,2\}, \{2,3\}, \{3,4\}, \{1,3,4\}\}$.

(a) Draw a Hasse diagram for the poset $(A, \subseteq)$.

(b) List all maximal elements of $(A, \subseteq)$.

(c) List all minimal elements of $(A, \subseteq)$.

(d) Are there the greatest and the least elements in $(A, \subseteq)$.

(e) Find the least upper bound and the greatest lower bound for A in the poset $(\mathcal{P}(\mathbb{Z}), \subseteq)$, if any.