## M E T U Department of Mathematics

Group		Fundamentals of Mathematics						
	Final							
Code	: Math	h 111		L	ast Name	:		
Acad. Year $$	: 2013	: 2013			Name : Student N			lo. :
Semester	: Fall		Timo chia	- D	Department : Section			;
Instructor : G.Ercan, S.Finashin, M.Kuzuanočku Ö.Kücükachalla					Signature :			·
Date : January 21, 2014					gnature	•		
Time	: 13:30				6 QUESTIONS ON 4 PAGES			
Duration	Duration : 120 minutes				80 TOTAL POINTS			
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1. (16pts) Give an example of a pair of sets A and B such that

• A is countably infinite and A - B is finite.

• B and A - B are both countably infinite.

• A, B and A - B are all uncountable.

• A and B are both uncountable but A - B is countable.

**2.** (12pts) Let A, B, C and D be sets. Suppose that  $A \sim B$  and  $C \sim D$ . Prove that  $A \times C \sim B \times D$ .

**3.** (12pts) Prove that the following formula holds for all  $n \in \mathbb{N}$ .

$$1^{3} + 2^{3} + 3^{3} + \ldots + n^{3} = \frac{n^{2}(n+1)^{2}}{4}.$$

4. (12pts) Prove that  $2^{2n} - 1$  is divisible by 3 for all natural numbers  $n \in \mathbb{N}$ .

5. (12pts) Let  $f: A \to B$  and  $X \subseteq A$ . If f is bijective then prove that f(A - X) = B - f(X).

- 6. (16pts) Let  $T = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x = y \text{ or } x^2 + y^2 = 1\}.$ 
  - Sketch the set T in the xy-plane.

• Is T a function from  $\mathbb{R}$  to  $\mathbb{R}$ ?

• Is T a partial order on  $\mathbb{R}$ ?

• Is T an equivalence relation on  $\mathbb{R}$ ?