# M ET U <br> Department of Mathematics 

| Group | Fundan | mentals of Mathematics Final |  | List No. |
| :---: | :---: | :---: | :---: | :---: |
| Code | : Math 111 | Last Name <br> Name <br> Department <br> Signature | Student No. <br> Section |  |
| Acad. Year | : 2013 |  |  |  |
| Semester | : Fall |  |  |  |
| Instructor | : G.Ercan, S.Finashin |  |  |  |
| M.Kиzucu | uoğlu, Ö.Kücüksakallı. |  |  |  |
| Date Time | : January 21, 2014 : 13:30 | 6 QUESTIONS ON 4 PAGES 80 TOTAL POINTS |  |  |
| Duration | : 120 minutes |  |  |  |  |  |
| ${ }^{2}$ | $\mathrm{l}^{4} \mathrm{l}^{5}{ }^{6}$ |  |  |  |

1. (16pts) Give an example of a pair of sets $A$ and $B$ such that

- $A$ is countably infinite and $A-B$ is finite.
- $B$ and $A-B$ are both countably infinite.
- $A, B$ and $A-B$ are all uncountable.
- $A$ and $B$ are both uncountable but $A-B$ is countable.

2. (12pts) Let $A, B, C$ and $D$ be sets. Suppose that $A \sim B$ and $C \sim D$. Prove that $A \times C \sim B \times D$.
3. (12pts) Prove that the following formula holds for all $n \in \mathbb{N}$.

$$
1^{3}+2^{3}+3^{3}+\ldots+n^{3}=\frac{n^{2}(n+1)^{2}}{4}
$$

4. (12pts) Prove that $2^{2 n}-1$ is divisible by 3 for all natural numbers $n \in \mathbb{N}$.
5. (12pts) Let $f: A \rightarrow B$ and $X \subseteq A$. If $f$ is bijective then prove that $f(A-X)=B-f(X)$.
6. (16pts) Let $T=\left\{(x, y) \in \mathbb{R} \times \mathbb{R}: x=y\right.$ or $\left.x^{2}+y^{2}=1\right\}$.

- Sketch the set $T$ in the $x y$-plane.
- Is $T$ a function from $\mathbb{R}$ to $\mathbb{R}$ ?
- Is $T$ a partial order on $\mathbb{R}$ ?
- Is $T$ an equivalence relation on $\mathbb{R}$ ?

