METU MATHEMATICS DEPARTMENT
MATH 111 - MAKEUP EXAM - FALL 2013

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Question 1. (20pts) Determine which of the following sets are countable. Explain briefly.

- $A = \mathbb{Z} \times \mathbb{Q}$.
- $B = \{(a, b) : a \in \mathbb{Q}, b \in \mathbb{R} - \mathbb{Q}\}$.
- $C = \mathcal{P}(\mathbb{R})$, the power set of $\mathbb{R}$.
- $D = \{A \subseteq \mathbb{N} : |A| > 1\}$.
- $E = \{A \subseteq \mathbb{N} : |A| \leq 1\}$.

Question 2. (12pts) Prove that $2^n > n^3$ for every natural number $n \geq 10$.

Question 3. (12pts) Suppose a sequence is defined as follows: $a_1 = 1$, $a_2 = 3$ and $a_k = 2a_{k-1} + a_{k-2}$ for all integers $k \geq 3$. Prove that a_n is odd for all natural numbers $n \in \mathbb{N}$ using induction.

Question 4. (12pts) Suppose A and B are sets and $f : A \to B$ is a function. Let R be an equivalence relation on B. Let S be the relation on A defined by

$$a_1 \; S \; a_2 \Longleftrightarrow f(a_1) \; R \; f(a_2)$$

Show that S is an equivalence relation on A.

Question 5. (12pts) Suppose R_1 and R_2 are partial orders on A. For each part, give either a proof or a counterexample to justify your answer.

- Must $R_1 \cap R_2$ be a partial order on A?
- Must $R_1 \cup R_2$ be a partial order on A?

Question 6. (12pts) Let A and B be sets. For each part, give either a proof or a counterexample to justify your answer.

- $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.
- $\mathcal{P}(A \times B) = \mathcal{P}(A) \times \mathcal{P}(B)$.