METU MATHEMATICS DEPARTMENT
MATH 111 - RESIT EXAM - FALL 2013

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Question 1. Let a, b and p be natural numbers such that $p \geq 2$. Suppose that if $p|ab$ then $p|a$ or $p|b$. Prove that $\sqrt{p}$ is an irrational number.

Question 2. Let $f : A \to B$ be a function. If Y_1 and Y_2 are subsets of B then show that $f^{-1}(Y_1 \cap Y_2) = f^{-1}(Y_1) \cap f^{-1}(Y_2)$.

Question 3. Let $S = \mathcal{F}(\{1, 2, 3\}, \mathbb{N})$ be the set of all functions from $\{1, 2, 3\}$ to $\mathbb{N}$. For $f, g \in S$, we write $f \preccurlyeq g$ if $f(x) \leq g(x)$ for every $x \in \{1, 2, 3\}$.

- Show that $(S, \preccurlyeq)$ is a partially ordered set.
- Is $\preccurlyeq$ a total order on S?
- Does S contain a maximal element?
- Does S contain a least element?

Question 4. Prove that $1^3 + 3^3 + \ldots + (2n - 1)^3 = n^2(2n^2 - 1)$ for all $n \in \mathbb{N}$.

Question 5. (15pts) Define E on $\mathbb{R}$ by $x\ E\ y \Leftrightarrow x - y \in \mathbb{Z}$.

- Show that E is an equivalence relation on $\mathbb{R}$.
- Show that $\mathbb{R}/E$ is uncountable.

Question 6. Let $\mathcal{F}(\mathbb{N}, \{0, 1\})$ be the set of all functions from $\mathbb{N}$ to $\{0, 1\}$ and let $\mathcal{P}(\mathbb{N})$ be the power set of $\mathbb{N}$. Prove that $\mathcal{F}(\mathbb{N}, \{0, 1\}) \sim \mathcal{P}(\mathbb{N})$.