MATH 112 Introductory Discrete Mathematics.
Suggested Exercise set II

Problem 1. If two integers are selected, at random and with out replacment, from $\{1, 2, \ldots, 100\}$ what is the probability the integers are consecutive?

Problem 2. If we selecte six numbers, without replacment, form $\{1, 2, \ldots, 100\}$ what is the probability that the second smallest number is 5?

Problem 3. A die is loaded so that the probability of a given number turns up is proportional to that number. So, for example, the outcome 4 is twice is likely as outcome 2, and outcome 3 is three time likely as outcome 1. The die is rolled two times. What is the probability the outcome is (a) 10; (b) at most 10; (c) double?

Problem 4. Let $x_1, x_2 \ldots, x_n$ be arbitrary integers. Show that $x_i + x_{i+1} + \cdots + x_{i+k}$ is divisible by n for some i and k, $i \geq 1$ and $k \geq 0$.

Problem 5. Show that among $n + 1$ distinct positive integers less than or equal to $2n$ there are two of them that are relatively prime.

Problem 6. Show that for arbitrary n there exist a number divisible by n that contains only the digits 7 and 0.

Problem 7. During six weeks Brace sends out at least one letter each day but no more than 60 letters in total. Show that there is a period of consecutive days during which he sends out exactly 23 letters.

Problem 8. What is the smallest value of n such that whenever $S \subset \mathbf{Z}_+$ and $|S| = n$, then there exist three elements x, y, $z \in S$ where all three have the same reminder upon division by $m \in \mathbf{Z}$.

Problem 9. Let S be a set of five positive integers the maximum of which at most 9. Prove that the sum of the elements in all nonempty subsets of S cannot all be distinct.

Problem 10. In how many ways can one arrange all of the letters in the word INFORMATION so that no pair of consecutive letters occurs more than once? [We want to count arrangments such as IINNOOFRMTA but not INFORINMOTA (where IN occurs twice) or NORTFNOIAMI (where NO occurs twice).]

Problem 11. An exam has 12 questions, whose total value is to be 200 points. In how many ways can we assing 200 if each question must count for at least 10, but no more than 25, points and the point value for each question is to be a multiple of 5?

Problem 12. If we roll a fair die five times, what is the probability that the sum of the five rolls is 20?

Problem 13. It is known that at the university 60 percent of the professors play tenis, 50 percent of them play bridge, 70 percent jog, 20 percent play tenis and bridge,30 percent play tenis and jog, and 40 percent play bridgr and jog. If some one claimed that 20 percent of the professors jog and play bridge and tenis, would you belive this claim? (Explaine).

Problem 14. If eight fair dice are rolled, what is the probability that all six numbers appear?

Problem 15. Find the number of ways to arrange the letters in LAPTOP so that none of the letters is in its original position.

Problem 16. In how many ways can we distribute 12 books to 12 students and then collect and redistibute the books so that that exactly six students have the same book twice.