EXERCISE SET III : RECCURENCE

Problem 1. *Find the general solution for each of the following relations.*
a) $4a_n - 5a_{n-1} = 0$, $n \geq 1$;
b) $3a_{n+1} - 4a_n = 0$, $n \geq 0$, $a_1 = 5$;
c) $2a_n - 3a_{n-1} = 0$, $n \geq 1$, $a_4 = 81$;
d) $a_n = 5a_{n-1} + 6a_{n-2}$, $n \geq 2$, $a_0 = 1$, $a_1 = 3$;
e) $3a_{n+1} = 2a_n + a_{n-1}$, $n \geq 1$, $a_0 = 7$, $a_1 = 3$;
f) $a_n - 6a_{n-1} + 9a_{n-2} = 0$, $n \geq 2$, $a_0 = 5$, $a_1 = 12$;
g) $a_{n+2} + a_n = 0$, $n \geq 0$, $a_0 = 0$, $a_1 = 3$;
h) $a_n + 2a_{n-1} + 2a_{n-2} = 0$, $n \geq 2$, $a_0 = 1$, $a_1 = 3$.
(The final answer should not involve complex numbers.)

Problem 2. *The number of bacteria in a culture is 1000 (approximately) and this number increases* 250% *every two hours. Use a recurrence relation to determine the number of bacteria after one day.*

Problem 3. *Laura invested* \$100 *at* 6% *interest compounded quarterly. How many months must she wait for her money to double ?*

Problem 4. *Motorcycles and compact cars can be parked in a row of* n *spaces, so that each motorcycle requires one space and each compact car needs two. Suppose that all cycles are identical in appearance, as are the cars. Find and solve a recurrence relation for the number of ways to park so that all the* n *spaces are filled up.*

Problem 5. *For* $n \geq 0$, *let* a_n *count the number of ways a sequence of* 1*'s and* 2*'s will sum up to* n. *(For example,* $a_3 = 3$, *because* 3 *can be presented in three ways:* $1 + 1 + 1$, $1 + 2$, *and* $2 + 1$.*) Find and solve a recurrence relation for* a_n.

Problem 6. *For* $n \geq 1$, *let* a_n *be the number of ways to present* n *as an ordered sum of positive integers, so that each summand is at least two. (For example,* $a_5 = 3$, *because* 5 *can be presented in three ways:* 5, $3 + 2$, *and* $2 + 3$.*) Find and solve a recurrence relation for* a_n.

Problem 7. *Start with one pair of rabbits and suppose that each pair produces one new pair in each of the next two generations and then dies. Find the number* f_n *of pairs belonging to the* n*-th generation.*

Problem 8. *The Lucas numbers* L_n *are defined by* $L_1 = 1$, $L_2 = 3$, $L_n = L_{n-1} + L_{n-2}$. *Obtain a formula for* L_n.

Problem 9. *Solve the recurrence relation* $a_{n+2} = a_n a_{n+1}$.

Problem 10. *Solve the recurrence relation* $a_{n+2}^2 - 5a_{n+1}^2 + 4a_n^2 = 0$*, where* $n \geq 0$*,* $a_0 = 4$*,* $a_1 = 13$*.*

Problem 11. *Determine the constants* b *and* c *if* $a_n = c_1 + c_2(7^n)$*,* $n \geq 0$*, is the general solution of the relation* $a_{n+2} + ba_{n+1} + ca_n = 0$*.*

Problem 12. *Solve each of the following recurrence relations.*

a) $a_{n+1} - 2a_n = 5$, $a_0 = 1$.
b) $a_{n+1} - 2a_n = 2^n$, $a_0 = 1$.
c) $a_{n+1} - a_n = 2n + 3$, $a_0 = 1$.
d) $a_{n+1} - a_n = 3n^2 - n$, $a_0 = 3$.
e) $a_n = 4a_{n-1} - 3a_{n-2} + 2^n$, $a_1 = 1$, $a_2 = 11$.
f) $a_{n+2} + 3a_{n+1} + 2a_n = 3^n$, $a_0 = 0$, $a_1 = 1$.
g) $a_{n+2} + 4a_{n+1} + 4a_n = 7$, $a_0 = 1$, $a_1 = 2$.
h) $a_{n+2} - 6a_{n+1} + 9a_n = 3(2^n) + 7(3^n)$, $a_0 = 1$, $a_1 = 4$.
i) $a_{n+3} - 3a_{n+2} + 3a_{n+1} - a_n = 3 + 5n$, $a_0 = 1$, $a_1 = 4$.
j) $a_{n+2}^2 - 5a_{n+1}^2 + 6a_n^2 = 7n$, $a_0 = a_1 = 1$.