EXERCISE SET IV : GRAPHS

Problem 1. *Graph Γ is a tree whose vertices are either tree-valent or leaves. What can be the number of leaves if the number of 3-valent vertices is 100.*

Problem 2. *(a) Find the number of edges in a graph Γ with 20 vertices, if 10 of these vertices have valency 3, 5 vertices have valency 5 and the rest have valency 7. (b) Assuming that graph Γ is planar, find the number of its regions and show that some regions should have degree 3 or less.*

Problem 3. *The regions of a planar graphs all have degree 4. Find the number of its edges and regions if the number of vertices is 100.*

Problem 4. *Prove that any tree is a bipartite graph.*

Problem 5. *Prove that in a bipartite planar graph the number of vertices is greater than the number of regions.*

Problem 6. *(a) Show that the regions of a planar graph Γ are two-colorable if the graph contains an Euler cycle. (b) Is it possible that Γ is two-colorable if it contains an Euler path which is not a cycle ?*

Problem 7. *Find the number of all spanning trees in (a) K_4; (b) $K_{2,3}$. Determine which of these trees are isomorphic.*

Problem 8. *Find the number of non-isomorphic types of (a) graphs with 3 vertices; (b) connected graphs with 4 vertices; (c) trees with 5 and 6 vertices.*

Problem 9. *Find (a) the chromatic numbers for all the graphs in the previous problem; (b) the edge-chromatic numbers for all these graphs.*

Problem 10. *Graph Γ has v vertices and e edges, where $v + e < 15$. Deduce that Γ is planar.*

Problem 11. *Describe a three-valent graph with $2n$ vertices which contains a Hamiltonian cycle.*

Problem 12. *Prove that graph $K_{n,m}$ is not Hamiltonian, unless $|n - m| \leq 1$.*

Problem 13. *Consider a three-valent bipartite graph Γ, whose vertices are colored in white and black (according to the partition). Prove that it has equal number of black and white vertices.*