# M E T U <br> Department of Mathematics 



READ THE PROBLEMS CAREFULLY AND GIVE DETAILED WORK

1. (15 pts) a) Find the number of monomials (monomial is a summand of the form $c x^{i} y^{j} z^{k}$, where $c$ is a constant) in the expansion of $(x+y+z+1)^{100}$.
b) Find the coefficient of $x^{8}$ in $\left(x^{2}+x+2\right)^{7}$.
c) How many telephone numbers can be formed with two digits 1 , two digits 2 , and three digits 3 , so that two (but not all three) digits 3 stand together ?
2. (15 pts) Suppose that a department contains 10 women and 6 men.
a) Three couples are married. In how many ways it could happen ?
b) How many ways are there to form a committee with 5 members if it must have more women than men?
c) How many ways are there to arrange all 16 department members in a row, so that no two men stand next to each other?
3. (15 pts) a) Four fair coins are tossed. Find the probability that at least three of them land heads up?
b) Suppose we flip a coin four times. What is the probability that exactly three heads occur given that at least one head occurred ?
c) What is more probable: rolling a total of 8 when two dice are rolled or rolling a total of 8 when three dice are rolled?
4. (15 pts) Two distinct integers $1 \leq a, b \leq 100$ are chosen at random.
a) What is the probability that one of them is not greater than 10 ? Estimate whether the probability is less or more than $20 \%$ ?
b) What is the probability that one of these two numbers is divisible by 3 , or 5 ?
