M E T U
Department of Mathematics

READ THE PROBLEMS CAREFULLY AND GIVE DETAILED WORK

1. (5+5+5 pts) An expansion presents $(x^2 + 2y - 3z + 2)^{20}$ as a sum of monomials of the form $c_{ijk}x^i y^j z^k$, where c_{ijk} is a coefficient.

a) Find the coefficient c_{435}.

b) Find the number of the coefficients c_{ijk}.

c) Find the sum of all the coefficients c_{ijk}.

2. (5+5+5 pts)a) In a class of 20 students two received grade AA, three grade BA and four grade BB. In how many ways it could happen ?

b) Eight men and four women should seat in a row of 12 chairs. In how many ways they can do it, so that no two women sit next to each other ?

c) Now the same people should take seats at a round table with the same condition. In how many ways they can do it ? (Two arrangements of people are considered to be the same if they differ by rotation.)

3. (6+4+5 pts) A fair die is rolled five times.

a) In how many ways one can obtain 16 as the sum of the five results ?

b) What is the probability to obtain sum 16 ?

c) Assume that a die is NOT fair, and 6 appears two times more often than any other number. What is the probability to obtain in sum 10 after rolling this die twice ?

4. (5 pts) After failing all his exams, John told to his parents that out of 100 freshman students at the department 60 failed Calculus, 55 failed Algebra, and 45 Geometry. Also, 30 failed both Calculus and Algebra, 25 Calculus and Geometry, and 20 Algebra and Geometry. There were 18 students who failed all three exams. Can it be a true ? (Explain.)

5. (4+6 pts) Suppose that 50 students failed one or more of the three exams.
a) Prove that one can find among them at least 17 students who failed the same number of exams.

b) Prove that one can find among them at least 8 students who all failed **and** passed exactly the same exams.