## M E T U Department of Mathematics

	Analytic Geometry					
	Midterm I					
Code Acad. Year Semester Instructors	: Math 115 : 2010-2011 : Fall : S.F, M.B, M.P.,	ath 115 010-2011 ill F, M.B, M.P., E.S	Last Name Name Department Signature	: : :	Student No Section	). : :
Date Time Duration	: November 11 2011 : 17:40 on : 90 minutes		4 QUESTIONS ON 4 PAGES TOTAL 60 POINTS			
1 2	3 4					

Show your work ! For the correct answers without any explanation you may receive no partial credits !

## Question 1 (5+5+5=15 points)

Consider a triangle with vertices A(1,1), B(3,5), C(-3,2).

a) Find the equation of line AB and its intersepts with the coordinate axes. What is the slope of this line ?

b) Find the equation of the line perpendicular to the side AB and passing through its midpoint.

c) Find a point D such that ABDC is a parallelogram (so that D is the opposite vertex to A). To which quadrant does point D belong ?

## Question 2(4+6+5=15 points)

Given equation  $r^2 = \sin 2\theta$  of a curve in polar coordinates

a) Determine if this curve is symmetric with respect to the polar axis ? Is it symmetric with respect to the pole ? (Do it not using the results of calculations and the sketch in part b).

b) Calculate the values of r for several values of  $\theta$  and use it to sketch the graph of this curve.

c) Find the equation of this curve in the Cartesian coordinates.

## Question 3 (4+6+5=15 points)

 $\overline{\text{Consider line } \ell \text{ defined by equation } x - y + 1} = 0$ 

a) Translate the xy-coordinate system horizontally, so that  $\ell$  passes through the origin of the new coordinate system. Find the equation of  $\ell$  in new coordinates  $\tilde{x}$  and  $\tilde{y}$ .

b) Rotate the xy-coordinate system so that  $\ell$  becomes parallel to the new axis  $\bar{x}$ . Find the equation of  $\ell$  in new coordinates  $\bar{x}$  and  $\bar{y}$  using the coordinate change formulas.

c) Point P has xy-coordinates (2, -4). Find its  $\tilde{x}\tilde{y}$  and  $\bar{x}\bar{y}$  coordinates.

a) Triangle ABC has vertices A=(1,0), B=(-1,2), and C=(4,1). Find  $\cos A$  and determine if angle A is greater or less than  $\frac{\pi}{2}$ ? (Use just your calculation, but not a sketch.)

b) Prove that a pair of vectors  $\vec{u}$  and  $\vec{v}$  are perpendicular if and only if their sum  $\vec{u} + \vec{v}$  and difference  $\vec{u} - \vec{v}$  have equal length.

c) Consider a triangle ABC and the midpoints D,F,E on its sides AB, AC, and BC respectively. Express vectors  $\overrightarrow{AE}$ ,  $\overrightarrow{BF}$ ,  $\overrightarrow{CD}$  in terms of vectors  $\overrightarrow{u} = \overrightarrow{AB}$ ,  $\overrightarrow{v} = \overrightarrow{AC}$ , and  $\overrightarrow{w} = \overrightarrow{BC}$ . Prove that  $\overrightarrow{AE} + \overrightarrow{BF} + \overrightarrow{CD} = \overrightarrow{0}$ .