# M E T U <br> Department of Mathematics 



Show your work! For the correct answers without any explanation you may receive no partial credits !

Question $1(5+5+5=15$ points)
$\overline{\text { Consider a triangle with vertices } A(1,1), B}(3,5), C(-3,2)$.
a) Find the equation of line AB and its intersepts with the coordinate axes. What is the slope of this line ?
b) Find the equation of the line perpendicular to the side AB and passing through its midpoint.
c) Find a point $D$ such that $A B D C$ is a parallelogram (so that $D$ is the opposite vertex to $A$ ). To which quadrant does point D belong ?

Question $2(4+6+5=15$ points)
Given equation $r^{2}=\sin 2 \theta$ of a curve in polar coordinates
a) Determine if this curve is symmetric with respect to the polar axis? Is it symmetric with respect to the pole ? (Do it not using the results of calculations and the sketch in part b).
b) Calculate the values of $r$ for several values of $\theta$ and use it to sketch the graph of this curve.
c) Find the equation of this curve in the Cartesian coordinates.

Question $3(4+6+5=15$ points)
Consider line $\ell$ defined by equation $x-y+1=0$
a) Translate the xy-coordinate system horizontally, so that $\ell$ passes through the origin of the new coordinate system. Find the equation of $\ell$ in new coordinates $\tilde{x}$ and $\tilde{y}$.
b) Rotate the xy-coordinate system so that $\ell$ becomes parallel to the new axis $\bar{x}$. Find the equation of $\ell$ in new coordinates $\bar{x}$ and $\bar{y}$ using the coordinate change formulas.
c) Point $P$ has $x y$-coordinates $(2,-4)$. Find its $\tilde{x} \tilde{y}$ and $\bar{x} \bar{y}$ coordinates.
a) Triangle ABC has vertices $\mathrm{A}=(1,0), \mathrm{B}=(-1,2)$, and $\mathrm{C}=(4,1)$. Find $\cos A$ and determine if angle A is greater or less than $\frac{\pi}{2}$ ? (Use just your calculation, but not a sketch.)
b) Prove that a pair of vectors $\vec{u}$ and $\vec{v}$ are perpendicular if and only if their sum $\vec{u}+\vec{v}$ and difference $\vec{u}-\vec{v}$ have equal length.
c) Consider a triangle ABC and the midpoints $\mathrm{D}, \mathrm{F}, \mathrm{E}$ on its sides $\mathrm{AB}, \mathrm{AC}$, and BC respectively. Express vectors $\overrightarrow{A E}$, $\overrightarrow{B F}, \overrightarrow{C D}$ in terms of vectors $\vec{u}=\overrightarrow{A B}, \vec{v}=\overrightarrow{A C}$, and $\vec{w}=\overrightarrow{B C}$. Prove that $\overrightarrow{A E}+\overrightarrow{B F}+\overrightarrow{C D}=\overrightarrow{0}$.

