M E T U
Department of Mathematics

Show your work ! For the correct answers without any explanation you may receive no partial credits !

Question 1 (5+5+5 = 15 points)

a) Find parametric equations for the line through $(1, 2, 3)$ that is perpendicular to the plane $2x - y + 3z + 115 = 0$.
At which points does this line intersect the given plane and the coordinate planes?

b) Find parametric equations of a line passing through $P(3, -2, -4)$, parallel to the plane $3x - 2y - 3z - 7 = 0$ and intersecting the line $\frac{x-2}{3} = \frac{y+4}{-2} = \frac{z-1}{2}$.

c)Find the symmetric partner of the origin with respect to the plane passing through the points $(1, 1, 0)$, $(1, 2, 3)$ and $(0, 1, 0)$.

Question 2 (5+5+5=15 points)

a) Find an equation of the plane containing the line $l = \{(x, y, z) : (x, y, z) = (t, 2t - 1, -t + 2)\}$ and parallel to the x-axis.

b) Find an equation for the plane consisting of all points that are equidistant from the points $(1, 1, 0)$ and $(0, 1, 1)$

c)Find the acute angle between the planes $x + y = 0$ and $y + z = 0$. Also find a vector equation for the line of intersection of these planes.

Question 3 (5+5+5=15 points)

a)Find the distances from the points $P(6, -1, -3)$ and $Q(-4, -2, 1)$ to the plane $x - 2y + 2z - 1 = 0$. Does the line through P and Q intersect the given plane?

b)Show that the line $x - 2 = \frac{y+3}{2} = \frac{z-1}{-4}$ is parallel to the plane $2y + z = 1$. What is the distance between the line and the plane?

a) Show that the lines

$$L_1 : x = 1 + 2t, \ y = 3t, \ z = 2 - t$$

$$L_2 : x = -1 + t, \ y = 4 + t, \ z = 1 + 3t$$

are skew and find the distance between them.

Question 4 (5+5+5=15 points)

a) Find the equation of the directrix of the parabola whose vertex is at $(1, 1)$ and whose focus is at $(3, -2)$.

b) Find the focus, directrix, vertex, axis and the length of the latus rectum of the parabola $y^2 + 10x = 0$.

c) Let $A(0, 0)$ and $B(3, 0)$. Prove that the set of points P, such that $|PA| = 2|PB|$ is a circle. Find its center and the radius.