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METU MATH 116, Midterm 2

Tuesday, May 3, 2011, at 17:40 (100 minutes), totally 60 points

Instructors: S.Finashin, S.Pamuk, D.Pierce, E.Solak

Instructions: Please, be accurate and show clearly the logic of your solutions. Only the answers are not enough: indicate your calculations and arguments.

Problem 1. (9 pts) Of the set $\{1, 2, 3, 4, 5, 6, 7, 9\}$, let $\sigma \in S_9$ be the permutation

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 7 & 4 & 1 & 6 & 8 & 2 & 9 & 5 \end{bmatrix}.$$

(a) Write σ as a product of disjoint cycles.

(b) Find σ^{100}. (Hint: find the order of σ).

(c) Determine whether permutation σ is even or odd.

Problem 2. (6 pts) Consider transpositions $\alpha = (12)$, $\beta = (23)$, $\gamma = (34)$ in S_4.

(a) Find the product $\alpha\beta\gamma$.

(b) Find $g \in S_4$ such that $\beta = g\alpha g^{-1}$.

Problem 3. (8 pts)

(a) Find the number of cosets for subgroups $3\mathbb{Z} \subset \mathbb{Z}$ and $\mathbb{Z} \subset \mathbb{Q}$. In the first case, give an explicit list of these cosets.

(b) Find the group multiplication table of $\mathbb{Z}/3\mathbb{Z}$ (pay attention that the group operation is addition).

(c) Prove that every element of $\mathbb{Q}/\mathbb{Z}$ has finite order.

Problem 4. (7 pts) Assume that G is a group of prime order p.

(a) What can be the order of an element $g \in G$? (Explain !)

(b) Prove that G is a cyclic group.

Problem 5. (10 pts) Suppose that H and N are subgroups of a group G. Assume that subgroup N is normal, and let $K = H \cap N$.

(a) Prove that K is a normal subgroup of H.

(b) Let $A = \{x \in G \colon xK = Kx\}$. Prove that $H \subseteq A$.

(c) Prove that A is a subgroup of G.

Problem 6. (5 pts) Let H and K be normal subgroups of G such that G/H has order 5, and G/K has order 3. Prove that for any $g \in G$, $g^{15} \in K \cap H$.

Problem 7. (6 pts) Consider subsets $4\mathbb{Z} \subset \mathbb{Z}$, $\{0\} \subset \mathbb{Z}$, $\varnothing \subset \mathbb{Z}$, $\mathbb{Z} \subset \mathbb{Q}$, $(0, \infty) \subset \mathbb{R}$, $\{[0], [1], [2], [4]\} \subset \mathbb{Z}_6$, $M_2(\mathbb{Z}) \subset M_2(\mathbb{R})$.

(a) Which of the above examples are NOT subrings? Why they are not subrings ?

(b) Which ones are NOT ideals? Why not ?

Problem 8. (9 pts) In each of the following two subrings of $\mathbb{Z}_{12}$, find a unity or show that there is no unity (multiplicative identity).

(a) $2\mathbb{Z}_{12} = \{[0], [2], [4], [6], [8], [10]\}$

(b) $3\mathbb{Z}_{12} = \{[0], [3], [6], [9]\}$

(c) Find all the zero divisors in the ring $\mathbb{Z}_{12}$.