Name:

Student number:

1	
2	
3	
4	
Σ	

METU MATH 116, Resit Exam

June 17, 2015, at 14:30 (100 minutes), totally 80 points

Instructors: M.Bhupal, S.Finashin, F.Özbudak, E.Solak

Instructions: Please, be accurate and show clearly the logic of your solutions. Only the answers are not enough: indicate your calculations and arguments.

Problem 1. (15 pts) Let R be a ring and $I \subset R$ its ideal. Prove that the set

$$K_I = \{x \in R \,|\, xa = 0 \text{ for all } a \in I\}$$

is also an ideal in R.

Problem 2. (20 pts) Let $I \subset \mathbb{Z}[x]$ be the principal ideal generated by polynomial $x \in R[x]$, namely, $I = (x) = \{xf(x) \mid f(x) \in \mathbb{Z}[x]\}$.
(a) Find and write explicitly an isomorphism $\mathbb{Z} \to \mathbb{Z}[x]/I$.

(b) Write explicitly the inverse map $\mathbb{Z}[x]/I \to \mathbb{Z}$ to the above isomorphism.

(c) For the principle ideal $J = (x^2 + x) \subset \mathbb{Z}[x]$, show that the quotient ring $R = \mathbb{Z}[x]/J$ contains zero divisors.

Problem 3. (10 pts) Give an example of a polynomial $f(x) \in \mathbb{R}[x]$ of degree 4 over field $\mathbb{R}$, such that $f(x)$ is reducible, but have no real roots.

Problem 4. (15 pts) Write $f(x) = 3x^4 + 5x^3 + x^2 + 5x - 2$ as a product of irreducible polynomials over $\mathbb{Q}$.

Problem 5. (20 pts) Given polynomial $f(x) = x^5 - x$
(a) present $f(x)$ as a product of irreducible factors over $\mathbb{Z}_5$;

(b) find the roots of $f(x)$ in $\mathbb{Q}$;

(c) find the complex roots of $f(x)$.