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METU MATH 116, Makeup

Tuesday, June 10, 2015, at 11:40 (100 minutes), totally 60 points

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Instructions: Please, be accurate and show clearly the logic of your solutions. Only the answers are not enough: indicate your calculations and arguments.

Problem 1. (15 pts) Solve a lnear system in $\mathbb{Z}_{13}$.

$$\begin{aligned} [6]x + [2]y &= [1] \\ [5]x + y &= [3] \end{aligned}$$

Problem 2. (15 pts) Assume that G is a group and $H \subset G$ its subgroup.
(a) Prove that subgroup H is necessarily normal if it has index $[G : H] = 2$.

(b) Give an example of a subgroup H of index $[G : H] = 3$ which is not normal. Justify that it is not normal.

Problem 3. (15 pts) Consider the ring of upper triangular matrices $T_2 = \{\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \mid a, b, c \in \mathbb{Z}\}$ and let $I = \{\begin{bmatrix} 0 & x \\ 0 & y \end{bmatrix} \mid x, y \in \mathbb{Z}\}$.

(a) Verify that I is a (2-sided) ideal in T_2.

(b) Prove that T_2/I is isomorphic to $\mathbb{Z}$. Write explicitly a map $T_2/I \to \mathbb{Z}$ and verify that it is a ring isomorphism.

Problem 4. (15 pts) (a) Present polynomial $f(x) = x^4 + x^3 + x^2 + x$ as a product of irreducible polynomials over $\mathbb{Z}_3$. Explain, why your factors are irreducible.

(b) Show that polynomial $f(x) = 2x^3 + x^2 + x + 1$ is irreducible over $\mathbb{Q}$.