M E T U Department of Mathematics

Group	Calculus with Analytic Geometry								List No.
	Midterm I								
Code	: Math 1.	19		Last N	ame	:			
Acad. Year	: 2009-2010 : Spring	/10		Name		:	Stud	Student No.	
Semester	· Spring			Depart	ment	:	Secti	on	:
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Date Time Duration	: April 10th 2010 : 13:30 : 110 minutes	Oth 2010 nutes		9 QUESTIONS ON 4 PAGES TOTAL 60 POINTS					3
1 2	3 4	5	6	7	8	9			

SHOW DETAILED WORK IN EVERY PROBLEM.

Question 1 (8 pts.) Find the following limits. Do not use the l'Hospitals rule! a) $\lim_{x\to\infty} \frac{\sin x - x}{x}$

b)
$$\lim_{x\to 0} \frac{1-\sqrt{(x^2+1)}}{x^2}$$

Question 2 (6 pts.) Find the equation of the tangent line to the curve $x \cos x + \sin y = \frac{1}{2}$ at the point $(\pi/2, \pi/6)$.

Question 3 (6 pts.) Let y = y(x) be a function of x. Find y'' if $x^2 + y^2 + x = 0$.

Question 4 (8 pts.) $f(x) = \begin{cases} mx^3 & x \le 1 \\ ax+b & x > 1 \end{cases}$ **a)** Find the relation between a, b and m for f to be continuous everywhere.

b) Let f be as above. If f(2) = 4 then determine the value of a, b and m for f to be differentiable everywhere.

Question 5 (6 pts.) A point is moving on the curve $x^3 + y^3 - 6xy = 0$. Its *x*-coordinate is increasing at a rate of 1 ($\frac{dx}{dt} = 1$) at x = 2. Find the change in the *y*-coordinate at that instant.

Question 6 (6 pts.) Using derivatives find the point on the line segment 2x + y = 1 ($0 \le x \le 1$) that is closest to the point (3, 1).

Question 7 (4 pts.) Evaluate $\frac{d}{dx}(\int_{x^2}^1 \cos^2 t \, dt)$.

Question 8 (4 pts.) Evaluate the integral $\int_0^{2\pi} (\sin 2x + x^3 - 5) dx$.

Question 9 (12 pts.) Let f(x) be the function given by $\frac{1}{1+x^3}$.

a) Determine the asymptote(s).

b) Determine the intervals of increase and decrease.

c) Determine the concavity and inflection points.

d) Sketch the graph of f.