# M ETU <br> Department of Mathematics 



## SHOW DETAILED WORK IN EVERY PROBLEM.

Question 1 ( 8 pts.) Find the following limits. Do not use the l'Hospitals rule!
a) $\lim _{x \rightarrow \infty} \frac{\sin x-x}{x}$
b) $\lim _{x \rightarrow 0} \frac{1-\sqrt{\left(x^{2}+1\right)}}{x^{2}}$

Question 2 ( $6 \mathbf{p t s}$.$) Find the equation of the tangent line to the curve x \cos x+\sin y=\frac{1}{2}$ at the point $(\pi / 2, \pi / 6)$.

Question 3 ( $6 \mathbf{p t s}$.) Let $y=y(x)$ be a function of $x$. Find $y^{\prime \prime}$ if $x^{2}+y^{2}+x=0$.

Question 4 (8 pts.) $f(x)=\left\{\begin{array}{ll}m x^{3} & x \leq 1 \\ a x+b & x>1\end{array}\right.$ a) Find the relation between $a, b$ and $m$ for $f$ to be continuous everywhere.
b) Let $f$ be as above. If $f(2)=4$ then determine the value of $a, b$ and $m$ for $f$ to be differentiable everywhere.

Question 5 ( 6 pts.) A point is moving on the curve $x^{3}+y^{3}-6 x y=0$. Its $x$-coordinate is increasing at a rate of $1\left(\frac{d x}{d t}=1\right)$ at $x=2$. Find the change in the $y$-coordinate at that instant.

Question 6 ( $6 \mathbf{p t s}$.$) Using derivatives find the point on the line segment 2 x+y=1(0 \leq x \leq 1)$ that is closest to the point $(3,1)$.

Question 7 (4 pts.) Evaluate $\frac{d}{d x}\left(\int_{x^{2}}^{1} \cos ^{2} t d t\right)$.

Question 8 (4 pts.) Evaluate the integral $\int_{0}^{2 \pi}\left(\sin 2 x+x^{3}-5\right) d x$.

Question 9 (12 pts.) Let $f(x)$ be the function given by $\frac{1}{1+x^{3}}$.
a) Determine the asymptote(s).
b) Determine the intervals of increase and decrease.
c) Determine the concavity and inflection points.
d) Sketch the graph of $f$.

