

Improper Integrals

① Direct computation and Comparison Test : as in the book..

Note Title

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\int_a^b

$f(x) dx$ s.t. the only singularity of f on $[a, b]$ is at b

or
(Abs conv \Rightarrow conv)

$$-|f(x)| \leq f(x) \leq |f(x)| \quad \forall x \in [a, b]$$

$$\therefore \int_a^b |f(x)| dx \text{ conv}, \text{ then so is } \int_a^b f(x) + |f(x)| dx \quad (1)$$

$$\int_a^b f(x) dx = \int_a^b f + |f| - |f| dx$$

$$\leq \underbrace{\int_a^b (f + |f|) dx}_{\substack{\text{true since} \\ \text{each of the} \\ \text{integral converges}}} - \underbrace{\int_a^b |f| dx}_{\substack{\text{by (1) above} \\ \text{conv}}} = \text{conv.}$$

∴ Thm: Abs conv \Rightarrow conv.

(Limit Comp. Test)

③ If the only singularity of f' & g' on $[a, b]$ is at b and

$$\lim_{x \rightarrow b^-} \frac{f(x)}{g(x)} = \begin{cases} 0 \\ \infty \\ 0 \neq L < \infty \end{cases}$$

Then

$$0 \Rightarrow \text{around } b, g(x) > f(x) \Rightarrow \int_a^b g(x) dx > \int_a^b f(x) dx > 0$$

$$\therefore \text{if } \int_a^b g(x) dx \text{ conv then so is } \int_a^b f(x) dx \\ \text{if } \int_a^b f(x) dx \text{ div } \dots \text{, } \int_a^b g(x) dx$$

$$\infty \Rightarrow \text{around } b, f(x) > g(x) \Rightarrow \int_a^b f(x) dx > \int_a^b g(x) dx > 0$$

$$\therefore \text{if } \int_a^b f(x) dx \text{ conv then so is } \int_a^b g(x) dx \\ \text{if } \int_a^b g(x) dx \text{ div } \dots \text{, } \int_a^b f(x) dx$$

$$L \Rightarrow \int_a^b f(x) dx \longleftrightarrow \int_a^b g(x) dx \quad (\text{ie either both conv. or both div.})$$

(note if both conv, this does NOT mean both conv. to the same number)