M E T U
Mathematics Department

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Math 260 – Basic Linear Algebra Final Exam						
2001–2002 **Fall Semester** **SF, HTK, EK, MP** **Monday, 14.1.2002** **13:00–15:30** **2.5 hours**	Name : Last Name : Signature :				Student Number : Section :	
	5 questions on 6 pages.					
Q.1 / 20	Q.2 / 20	Q.3 / 20	Q.4 / 20	Q.5 / 20		**Total** / **100**
					SHOW ALL YOUR WORK.	

Q. 1. A 3×3 real SYMMETRIC matrix A satisfies $A\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}2\\2\\2\end{bmatrix}$, $A\begin{bmatrix}1\\-2\\1\end{bmatrix}=\begin{bmatrix}-1\\2\\-1\end{bmatrix}$,

and $A\begin{bmatrix}1\\0\\-1\end{bmatrix}=\begin{bmatrix}1\\0\\-1\end{bmatrix}$.

(a) What are the eigenvalues, the trace, and the determinant of A?

(b) Find A.

Q. 2. **(a)** Find the eigenvalues and the eigenvectors of $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

(b) Is A diagonalizable? If yes, write a basis for $\mathbb{R}^3$ in which the matrix of the linear transformation represented by A is diagonal. If no, explain why not.

(c) Use the CAYLEY-HAMILTON theorem to find A^{-1}.

Q. 3. Let $f(x_1, x_2) = 19x_1^2 - 6x_1x_2 + 11x_2^2 - 2x_1 - 6x_2 - 3$.

(a) Determine the type of the curve $f(x_1, x_2) = 0$.

(b) Find an orthogonal coordinate-change matrix P that diagonalizes the quadratic part of f.

(c) Write down formulas that express the old coordinates x_1 and x_2 in terms of the new coordinates y_1 and y_2.

(d) Write the equation of the curve $f(x_1, x_2) = 0$ in the (y_1, y_2)-coordinates in standard form by COMPLETING THE SQUARE.

Q. 4. **(a)** Find all solutions to the system of linear equations $AX = B$, that is,

$$\begin{bmatrix} 1 & -2 & -3 & 5 \\ 1 & -2 & -2 & -1 \\ -2 & 4 & 7 & -16 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}.$$

(b) Considering A as the matrix of a linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ and using part **(a)**, find a basis for the kernel of T, find the nullity of T, find the rank of T, find a basis for the image of T, and determine whether or not T is one-to-one.

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Q. 5. Each part is a separate question independent of the other parts. Give explanations in the first three parts.

(a) Is the set $S = \{\, f \;:\; f$ is continuous on $[0,1]$ with $\int_0^1 f(t)\,dt \geq 0\,\}$ a vector space?

(b) Is the set of functions $F = \{e^{-x}, 0, e^{x}\}$ on the real line linearly independent?

(c) Is there a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ simultaneously satisfying $T(1,2) = (3,4)$, $T(5,6) = (7,8)$, and $T(6,8) = (10,14)$?

(d) Show that in a Euclidean (inner product) space if $\langle x+y, x-y\rangle = 0$, then $\|x\| = \|y\|$.

(e) Show that in a vector space if the set $A = \{u, v, w\}$ is linearly independent, then the set $B = \{u+v, v+w, w+u\}$ is also linearly independent.

(f) Let $E = \{e_1 = (1,0), e_2 = (0,1)\}$ be the standard basis and $F = \{f_1 = (1,2), f_2 = (3,4)\}$ be another basis for $\mathbb{R}^2$. Find the transition matrix Q from E to F. If the coordinates of a vector v is $(5,6)$ with respect to E, find the coordinates of v with respect to F BY USING Q.