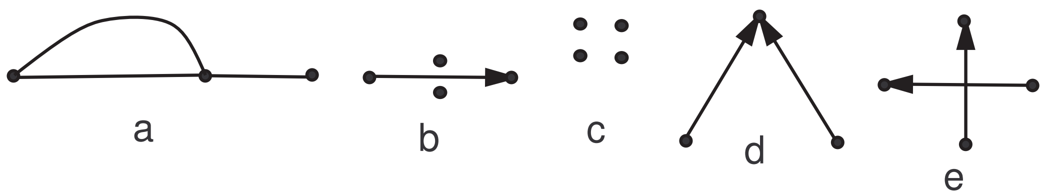
METU Department of Mathematics Math 341 Graph Theory Fall 2008

MAKEUP

Problem 1 (2pts). *Which of the following diagrams present*

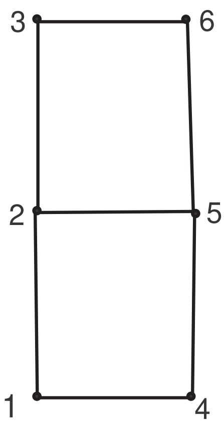
(1) *graphs*
(2) *multigraphs (not graphs)*
(3) *digraphs*



Problem 2(3pts). *Determine which ones among sequences a-e are*

(1) *walks but NOT trails*
(2) *trails but NOT paths*
(3) *closed walks but NOT cycles*
(4) *Eulerian trails*
(5) *Hamiltonian cycles.*

a) 12321; b) 123456; c) 123654: d) 23652145; e) 2365412



Problem 3(3 pts). *Sketch a planar diagram of graph G obtained from K_5 by removing one edge. Sketch a planar diagram of the dual graph G'.*

Problem 4(3 pts). *Give a definition of edge-connectivity (line-connectivity) number $\lambda(G)$ of graph G.*

Problem 5(3 pts). *Give the numbers v of vertices, e of edges, and f of faces for the 5 regular polyhedra (Platonic solids)*

(1) *tetrahedron*
(2) *cube*
(3) *octahedron*
(4) *dodecahedron*
(5) *icosahedron*

Problem 6(4 pts). *What is the minimal number of colors required to paint the vertices of*

(1) *a tetrahedron*
(2) *a cube*
(3) *an octahedron*
(4) *a dodecahedron*

so that adjacent vertices are colored differently ?

Problem 7(4 pts). *Give a list of matrices (if any) which are*

(1) *incidence matrices of some graphs*
(2) *incidence matrices of multi-graphs, but not of graphs*
(3) *adjacency matrices of some graphs*
(4) *adjacency matrices of directed graphs, but not of usual graphs.*

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, C = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, E = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

Problem 8(5 pts). *Find the following groups of symmetries*

(1) $\mathrm{Sym}(K_{5,6})$
(2) *complete symmetry group of a tetrahedron*
(3) *group of rotations of an octahedron*
(4) *group of rotation of a dodecahedron.*

Problem 9(5 pts). *Determine the order of the following groups*

(1) *group of symmetries of* $K_{4,4}$
(2) *group of symmetries of* $K_4 + K_4 + K_4$ *(the graph with 3 components* K_4*)*
(3) *complete group of symmetries of an icosahedron*
(4) *group of rotations of a football.*

Problem 10(4 pts).

(1) *Find the number of binary trees with a root and* 6 *leaves.*
(2) *Find the number of triangulations of a* 7*-gon ?*

Problem 11(4 pts). *Sketch a planar graph diagram of*

(1) *a truncated cube*
(2) *a tetrahedron with two truncated (opposite) edges*
(3) *a dodecahedron*

MAKEUP

PART II

Problem 12(5 pts). *Which relations do you know between the following number characterizing a graph G*

(1) *the minimal vertex degree $\delta(G)$ of G,*
(2) *the maximal vertex degree $\Delta(G)$*
(3) *connectivity $\kappa(G)$ of G*
(4) *the edge connectivity $\lambda(G)$*
(5) *the chromatic number $\chi(G)$ of G*
(6) *the edge chromatic number $\chi'(G)$.*

Problem 13(5 pts). *Formulate at least 4 different conditions characterizing blocks which has more than two vertices. Each condition should be about existence of certain paths or cycles.*

Problem 14(10 pts). *Determine the connectivity κ, the chromatic number χ, and the edge chromatic number χ' for*

(1) *tetrahedron-graph with one truncated vertex*
(2) *associahedron-graph*

Problem 15(10 pts). *Sketch a planar graph diagram of a permutohedron-graph so that*

(1) *the infinity region is quadrilateral*
(2) *the infinity region is hexagonal*

Sketch the dual planar diagram in the both cases.

Problem 16(5 pts). *Assume that Γ is a trivalent graph on a surface of genus 2 such that exactly half of the regions has degree 6 and another half has degree 7. Determine the number of vertices, regions and edges of Γ.*

Problem 17(5 pts). *Give a list of the graphs and digraphs on the figure below (if any) which have a given group of symmetry*

(1) $\mathbb{Z}_2$
(2) $\mathbb{Z}_2 \times \mathbb{Z}_2$
(3) D_4