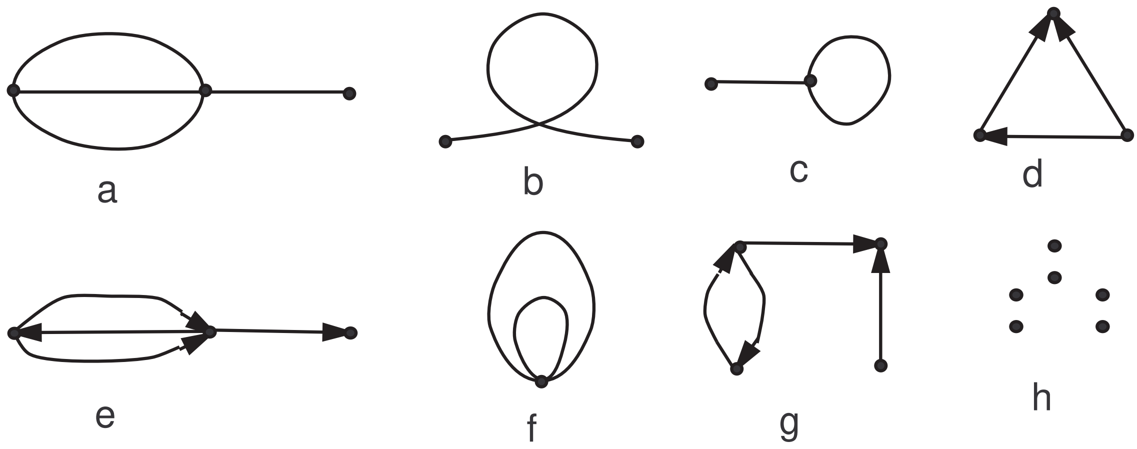
MIDTERM I (PART I)

Problem 1 (7pts). *Which of the following diagrams present*

(1) *graphs*
(2) *multigraphs*
(3) *pseudo-graphs*
(4) *digraphs*
(5) *oriented graphs*
(6) *directed multigraphs*
(7) *none of the above*

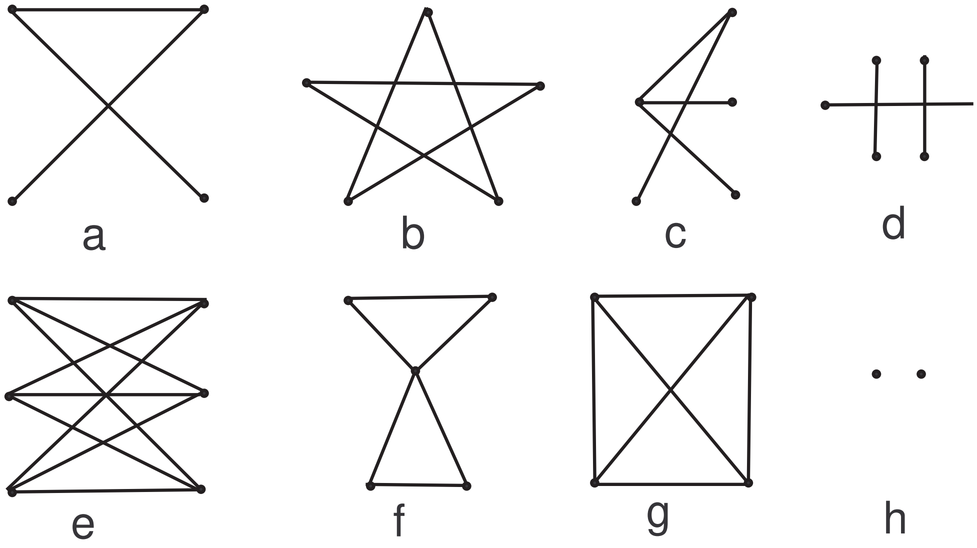


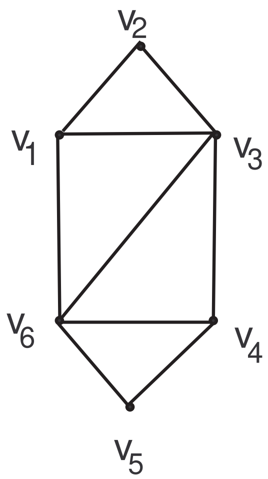
Problem 2. *Which one of the following graphs is*

(1) *a graph-path but not a cycle*
(2) *a graph-cycle*
(3) *a tree*
(4) *a forest*
(5) *a complete graph*
(6) *an Eulerian graph*
(7) *a bipartite graph.*

Problem 3. *Determine which ones of the following sequences of vertices in the graph sketched below are*

(1) *NOT walks*
(2) *closed walks*
(3) *walks but NOT trails*
(4) *Eulerian trails*
(5) *paths but NOT cycles*
(6) *Hamiltonian paths*
(7) *cycles.*





a) $v_1v_5v_1$
b) v_1v_3
c) $v_1v_3v_5v_1$
d) $v_1v_2v_4v_5$
e) $v_1v_2v_3v_4v_5$
f) $v_1v_2v_3v_1v_2v_3v_1$
g) $v_1v_3v_5v_1v_2$

Problem 4. *Find the number of edges in the following graphs*

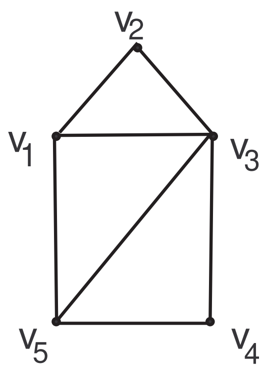
(1) K_{10},
(2) $K_{7,8}$,
(3) $\overline{K}_{5,6}$,
(4) *dodecahedron (which has 20 trivalent vertices).*

Problem 5. *What is the eccentricity of a vertex of*

(1) K_n, $n \geq 2$,
(2) $K_{p,q}$, $p, q \geq 2$,
(3) *the cube-graph (formed by the vertices and edges of a cube)*

Problem 6. *For the graph Γ shown below*

(1) *sketch the subgraph induced by vertices $\{v_1, v_3, v_4, v_5\}$;*
(2) *sketch three non-isomorphic spanning trees;*
(3) *find the dimension of the edge space $\mathcal{E}(\Gamma)$;*
(4) *find the dimension of the cycle space $C(\Gamma)$;*
(5) *determine if the sum of all the edges of Γ is a cycle ? (explain);*
(6) *present a set of basic cycles;*
(7) *describe an Euler's trail.*



Problem 7.

(1) *Sketch the line graph* $L(K_{2,3})$.
(2) *What are the degrees of vertices of the line graph* $L(K_{p,q})$ *?*
(3) *Prove that* $L(K_{p,q})$ *is not a bipartite graph for* $p, q \geq 2$*, except one case. Find the exceptional case and the corresponding graph* $L(K_{p,q})$.
(4) *Prove that* $K_{2,3}$ *is NOT a line graph* $L(\Gamma)$ *for some graph* Γ.

Problem 8. *Sketch the clique graph* (Γ) *for the graph* Γ *from the problem 6.*

Problem 9.

(1) *Find the characteristic sequence for the labelled tree sketched below.*


3
4
5

(2) *Find a tree with the following characteristic sequence* 443322115.