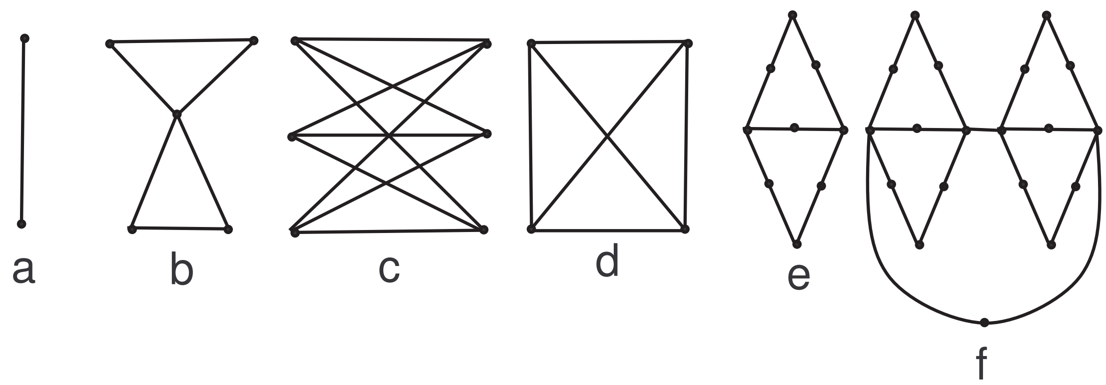
METU Department of Mathematics Math 341 Graph Theory Fall 2008

MIDTERM II

Problem 1(5 pts). *Which one(s) of the following graphs*

(1) *is a block*
(2) *has an Eulerian trail*
(3) *has an Eulerian closed trail*
(4) *has a Hamiltonian path*
(5) *has a Hamiltonian trail*



Problem 2(2 pts). *Write down the relation between the minimal vertex degree of a graph, its connectivity and its edge connectivity.*

Problem 3(3 pts). *Formulate Menger's theorem (on connectivity of a graph).*

Problem 4(5 pts). *What is the minimal number of colors required to paint the faces*

1. *of a tetrahedron*
2. *of a cube*
3. *of an octahedron*

so that faces with a common edge are colored differently ?

What is the minimal number of color that will be for sure enough to paint the faces of any polyhedron with n faces ?

Problem 5(2 pts). *Why every tree is 2-colorable ?*

Problem 6(2 pts). *Assume that the vertices of graph G have degrees either 3 or 4. What can you say about the edge chromatic number G ?*

Problem 7(1 pts). *Formulate Euler's formula for a polyhedron ?*

Problem 8(5 pts). *Give the numbers v of vertices, e of edges, and f of faces for the 5 regular polyhedra (Platonic solids)*

1. *tetrahedron*
2. *cube*
3. *octahedron*
4. *dodecahedron*
5. *icosahedron*

Problem 9(5 pts). *A polyhedron P has 20 triangular faces.*

1. *Determine the number e of its edges.*
2. *Determine the number v of its vertices.*
3. *Determine the numbers f', e', v' of faces, edges and vertices of the polyhedron P' dual to P.*

Midterm II – part II

Problem 10(4 pts). *Formulate at least 4 different conditions which are all equivalent to saying that graph G which has more than two vertices is a block.*

Problem 11(4 pts). *Prove that for any $n, m \geq 4$ the bipartite graphs $K_{n,m}$ has connectivity ≥ 4 ?*

Problem 12(10 pts). *Determine the connectivity κ, the chromatic number χ, and the edge chromatic number χ' for*

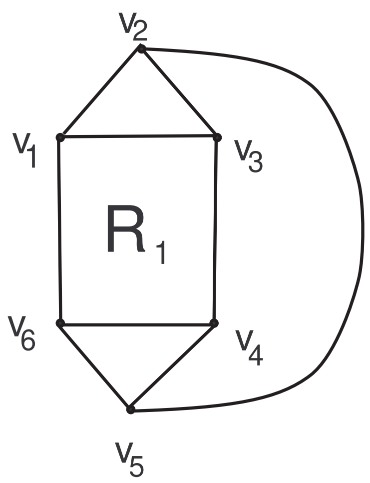
(1) *tetrahedron-graph*
(2) *cube-graph*
(3) *octahedron-graph*

Give your arguments. Formulate your guess (without giving arguments) about κ, χ, and χ' in the case of dodecahedron and icosahedron.

Problem 13(5 pts). *Construct an example of a trivalent graph with connectivity 2 and edge connectivity 3.*

Problem 14(4 pts). *For the graph Γ shown below*

(1) *sketch the dual graph*
(2) *sketch a diagram in which region R_1 becomes the infinity region*



Problem 15(3 pts). *Describe the 3-coloring of the edges corresponding to the 4-coloring of the map shown below.*