METU Department of Mathematics Math 422 Geometric Topology Spring 2014

MIDTERM 2 MAY 7 (60 points in 6 problems)

Name and the student number:

Problem 1. (14pts) *(a) State Euler's formula for graphs on surfaces. Under which condition on the graph this formula is satisfied ?*

(b) Give a definition of simplicial complex.

(c) What is a chain complex ?

(d) What are the homology groups of a chain complex ?

(e) How the boundary map of a simplicial chain complex is defined ?

(f) What are the homology groups of surfaces F_g and N_k ?

(g) Which Δ-triangulation of a surface is a simplicial triangulation ?

Problem 2. (5pts) *For the link diagram sketched below*
(a) find the topological type of its span,

(b) sketch a graph being a deformational retract of this span.

Problem 3. (10pts) *Consider surface F glued from a hexagon according to the word $abacb^{-1}c^{-1}$. Determine if the following curves on the hexagonal model are one-sided or two-sided.*

(a) Line segment connecting the midpoints of the sides "a".

(b) Line segment connecting the midpoints of the sides "b".

(c) The diagonal separating sides ab from $acb^{-1}c^{-1}$.

(d) The side a.

(e) The side b.

Problem 4. (4pts) *Consider a graph with vertices A, B, C and edges $[AB]$, $[BC]$¡ and $[AC]$ as a simplicial complex, C.*
(a) For $x = 2[AB] - 3[BC] + [AC]$ find $\partial_1 x$.

(b) Give examples of a cycle in $C_1(C)$ and a boundary in $C_0(C)$.

Problem 5. (12pts) *Consider surface F obtained from a hexagon ABCDEF by gluing side AB to DE and BC to FE. Divide the hexagon into triangles by diagonals AC, CE, and CF.*

(a) Is it a simplicial triangulation, or Δ-triangulation of F ? (Explain.)

(b) What are the generators of the chain groups C_0, C_1 and C_2 ?

(c) Find the boundary of the chain $2[ABC] - [CDE]$.

(d) Calculate the homology group $H_2(F)$ using this chain complex.

Problem 6. (15pts) *Consider a polygonal cell complex X, whose 2-cells are represented by words $abcb^{-1}$, $acda^{-1}$, bcb.*

(a) Describe its chain groups C_i, $i = 0, 1, 2$, and the boundary maps $\partial_2 \colon C_2 \to C_1$ and $\partial_1 \colon C_1 \to C_0$.

(b) Find the homology groups $H_i(X)$, $i = 0, 1, 2$.