Problems for Midterm I

1. Show that in the Poincare model, there exists precisely one hyperbolic line connecting any pair of points.
2. Show that there exisit precisely one Möbius transformation which sends three given points, $a, b, c \in \mathbb{R}$, to the points 0, 1, and ∞. Deduce that there exists precisely one Möbius transformation, which sends a, b, c to $x, y, z \in \mathbb{R}$.
3. Show that the hyperbolic plane is homogeneous and isotropic.
4. Show that the inversions, Möbius transformations, and stereographic projections are conformal.
5. Prove that an inversion maps circles and straight lines into circles or straight lines.
6. Prove that Möbius transformations preserve the cross-ratio.