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Σ	

Name:

Student number:

METU MATH 476, Final

Wednesday, June 6, 2012, at 15:00 (100 minutes), totally 70 points (+25 points for take-home)

Instructor: S.Finashin

Problem 1. (10 pts) (a) Find equations f_t of the pencil of conics, which contains $f_0 = xy$ and $f_1 = (x-1)^2 + (y-1)^2 - 1$. Sketch several conics A_t of this pencil making clear that their union is the whole plane. In particular, find the equation and sketch the conic f_t passing through the point $(-1, 1)$.

(b) Which of the conics f_t are singular ? Find equations and sketch.

(c) Find the intersection 0-cycle $A_0 \centerdot A_1$.

Problem 2. (10 pts) A sextic A has 5 nodes (non-degenerated double points) and 2 cusps. What is the genus of the normalization, $\tilde{A}$, of A.

Problem 3. (10 pts) A rational function $f : \Sigma \to P^1$ has degree 4. Its branching locus consists of double ramification points P_1, P_2, P_3, and a triple ramifications point Q.

(a) Find the ramification divisor of f.

(b) Find the genus of Σ.

Problem 4. (10 pts) Describe normalization by blowing up of the singularity $x^7 = y^3$. Sketch the curve on each of the steps of resolution.

Problem 5. (10 pts) Consider some points P_k, $k = 1, 2, 3$, on a Riemann surface A. Describe the functions f which belong to $L(D)$ and the 1-forms ω which belong to $K^1(D)$ for $D = 2P_1 + 3P_2 - P_3$ (in terms of the multiplicities of the poles and zeros at points P_k).

Problem 6. (20 pts) Let P denote a point on a curve A of genus $g = 4$. Suppose that the canonical class divisors on A contains a multiple of some point, mP, $m \geq 0$.
(a) Justify that $m = 6$.

(b) Write down the Riemann-Roch theorem applying it to the divisors nP, $n = 1, 2, 3, \ldots$.

(c) Using the results of (a), find all possible sequences $\ell(P), \ell(2P), \ell(3P), \ell(4P), \ell(5P), \ell(6P), \ldots$.

(d) Find the corresponding sequences $i(P), i(2P), i(3P), \ldots$ for each of the sequence in (b). (Recall that $i(D) = \dim K^1(D)$.)