Name:

Student number:

METU MATH 476, Homework 1 Tuesday, April 3, 2012 Instructor: S.Finashin

Problem 1. Parameterize affine curve $x^5 + y^4 = 0$ and explain why it is rational.

Problem 2. (a) Find affine coordinate system (after appropriate linear projective change of coordinates) in which hyperbola $x^2 - y^2 = 1$ becomes parabola.

(b) Which line (in terms of xy-coordinates) becomes the infinity line of the new affine coordinate system ?

(c) Which point (in terms of the initial projective coordinates) lies at infinity in the new coordinate system ?

Problem 3. Is curve xy = 1 reducible ? Explain.

Problem 4. Is projection $C \to \mathbb{A}$ from a hyperbola $C = \{xy = 1\}$ to x-coordinate line \mathbb{A} a biregular map? Is it birational? Explain.

Problem 5. Consider the cross-ratio t = [a, b; c, d]. Which other values of the cross-ratio can be obtained if we permute the order of a, b, c, d in all possible ways ?

Problem 6. Find a projective transformation of $P^1_{\mathbb{R}}$ sending points 1, 2, 3 into 0, 1, ∞ .

Problem 7. (a) Find the (real and complex) singular points of cubic $x^3 + y^3 + z^3 - 3xyz = 0$, if any.

(b) Show that this cubic is in fact a union of three lines (Hint: use Bezout theorem).

Problem 8. Find a theorem applying to the Pascal theorem the polar duality with respect to the conic involved into the Pascal theorem.