Name:

Student number:

## METU MATH 476, Homework 1

Tuesday, April 3, 2012
Instructor: S.Finashin

Problem 1. Parameterize affine curve $x^{5}+y^{4}=0$ and explain why it is rational.

Problem 2. (a) Find affine coordinate system (after appropriate linear projective change of coordinates) in which hyperbola $x^{2}-y^{2}=1$ becomes parabola.
(b) Which line (in terms of $x y$-coordinates) becomes the infinity line of the new affine coordinate system?
(c) Which point (in terms of the initial projective coordinates) lies at infinity in the new coordinate system?

Problem 3. Is curve $x y=1$ reducible ? Explain.

Problem 4. Is projection $C \rightarrow \mathbb{A}$ from a hyperbola $C=\{x y=1\}$ to $x$-coordinate line $\mathbb{A}$ a biregular map? Is it birational ? Explain.

Problem 5. Consider the cross-ratio $t=[a, b ; c, d]$. Which other values of the cross-ratio can be obtained if we permute the order of $a, b, c, d$ in all possible ways ?

Problem 6. Find a projective transformation of $P_{\mathbb{R}}^{1}$ sending points $1,2,3$ into $0,1, \infty$.

Problem 7. (a) Find the (real and complex) singular points of cubic $x^{3}+y^{3}+z^{3}-3 x y z=0$, if any.
(b) Show that this cubic is in fact a union of three lines (Hint: use Bezout theorem).

Problem 8. Find a theorem applying to the Pascal theorem the polar duality with respect to the conic involved into the Pascal theorem.

