

Name:

Student number:

## METU MATH 476, Midterm 2, Part 2 (Take-home)

May 10-11, 2012, totally 30 points

**Instructor:** S.Finashin

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**Problem 1. (5 pts)** A function  $f$  on the cubic  $A = \{y^2 = x^3 - 4x\}$  in the affine chart  $x, y$ , is defined as  $f(x, y) = xy$ . Analyze its zeros and poles (including the ones at infinity), and find the principal divisor  $(f)$  on the projectivization of  $A$ .

**Problem 2. (5 pts)** Consider a differential form  $\omega = x dy$  on the same curve  $A$ . Find its zeros and poles, in particular, at the infinity. Find the canonical divisor defined by  $\omega$ .

**Problem 3. (10 pts)** Find the index of the zero at the origin of the following differential forms in  $\mathbb{C}$ :

(a)  $\omega_1 = (x + y) dy + 2y dx$

(b)  $\omega_2 = \bar{z} dz.$

**Problem 4. (5 pts)** Prove that a quartic with four singular points is reducible.

**Problem 5. (5 pts)** Describe the blowup resolution process for the singularity  $y^3 = x^5$ . Sketch the position of the curve with respect to the exceptional curves at each step.