Name:

Student number:

1	
2	
3	
Σ	

METU MATH 476, Final, Part II (Take-home)

To return on Friday, June 8, 2012, before 15:00

Instructor: S.Finashin

Problem 1. (5 pts) Justify that $h = (x-1)^4 + (y-1)^4 - 1$ belongs to the ideal (f_0, f_1), where $f_0 = xy$ and $f_1 = (x-1)^2 + (y-1)^2 - 1$, by applying the fundamental Noether's theorem.

Problem 2. (5 pts) Two quintics A and B have both a cusp at a point P. The other intersection points, $P_1, \ldots, P_n$, are non-singular. What can be the values of n ?

Problem 3. (5 pts) Consider points $P_{\pm} = (-1, \pm\sqrt{3})$ on the curve $A = \{y^2 = x^3 - 4x\}$. Find a family of functions $f \in L(D)$ (including not only constants), where $D = P_+ + P_-$. Conclude that $\ell(D) \geq 2$.

Problem 4. (10 pts) Let A be the normalization of a quartic curve with one cuspidal singularity. Suppose that the canonical class divisors on A contains a multiple of some point, mP.
(a) Find m.

(b) Find all possible sequences $\ell(P), \ell(2P), \ell(3P), \ell(4P), \ell(5P), \ell(6P), \ldots$.

(c) Show that A is hyperelliptic by considering the projection $f : A \to P^1$ from the cusp of A.

(d) How many branch points of f are there ?

(e) Does f have a branch point at the cusp ?