Name:

Student number:

METU MATH 476, Midterm 1

Tuesday, April 3, 2012, at 17:40 (90 minutes), totally 40 points

Instructor: S.Finashin

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Problem 1. (10 pts) (a) Find the equation of conic $x^2 + xy - z^2 = 0$ in the three standard affine charts.

(b) Find the Newton polygon of the quartic with the affine equation $x^2 + y^2 + xy^3 = 0$.

(c) Which kind of singularities are there at the vertices $[0 : 0 : 1]$, $[0 : 1 : 0]$ and $[1 : 0 : 0]$?

Problem 2. (5 pts) Determine for which values of a (real or complex) the cubic $x^3 + y^3 + z^3 - axyz = 0$ is singular. Find a real singular point for some real value of a.

Problem 3. (5 pts) Find the polar map for the conic $C = \{x^2 + y^2 + 2xz = 0\}$, and the dual curve for C.

Problem 4. (10 pts) (a) Find the inflection points of cubic $A = \{x^3 + y^3 + z^3 = 0\}$ using its Hessian.

(b) Find the tangent lines to A at the inflection points.

(c) Find the tangent lines to A passing through $[1 : 0 : 0]$.

Problem 5. (10 pts) (a) Find the values of $[t : s] \in P^1$, for which the binary quadric $s(x + y)^2 + t(x^2 + y^2)$ is singular.

(b) Consider a pencil of binary cubics $h_{[t:s]}(x, y) = sf(x, y) + tg(x, y)$ (here f and g are homogeneous polynomials of degree 3). How many singular binary cubics are there in this pencil if the pencil is generic ? (**Hint:** in the space of binary cubics relate the pencil and the discriminant.)