Middle East Technical University Fall 2000 Math 537 Algebraic Topology I

MIDTERM II

DECEMBER 23, 2000

Problem 1. (15 pts) *Formulate a definition of*

(1) *a chain complex of abelian groups,*
(2) *the Betti numbers,*
(3) *a simplicial complex,*
(4) *a homotopy between chain maps* $f, g \colon C \to C'$.

Problem 2. (15 pts) *Find the homology groups of*

(1) *a punctured sphere,*
(2) *a punctured torus,*
(3) *a torus punctured in two points.*

Problem 3. (20 pts) $X = S^2/\{p_1, p_2, p_3\}$, *i.e.,* X *is obtained by collapsing a tree-point set on a sphere.*

(1) *Describe a cellular decomposition of* X.
(2) *Describe the corresponding cellular chain complex (groups* C_n *and the differentials* d_n*).*
(3) *Compute the cellular homology groups,* $H_*(X)$.

Problem 4. (20 pts) *Consider a set* $X \subset S^4$ *homeomorphic to* $T^2/\{p_1, p_2\}$ *(the quotient of a torus by a two-point set). Determine the groups*

(1) $H_*(X)$,
(2) $X_*(S^4/X)$.

Problem 5. (20 pts) *A cubic curve* $A \subset \mathbb{C}\mathrm{P}^2$ *is known to be a torus, such that the generator* $[A] \in H_2(A) \cong \mathbb{Z}$ *maps into* $3 \subset \mathbb{Z} \cong H_2(\mathbb{C}\mathrm{P}^2)$ *under the inclusion homomorphism* $i_* \colon H_2(A) \to H_2(\mathbb{C}\mathrm{P}^2)$. *Find the groups* $H_*(\mathbb{C}\mathrm{P}^2, A)$.

Problem 6. (20 pts) *Determine the groups* $H_*(S \smallsetminus K)$, *where* $K \subset S^3$ *is a knot (i.e.,* $K \cong S^1$*), using a Mayer-Vietoris exact sequence.* **Hint:** *use a splitting* $S^3 = T \cup N$, *where* T *is a solid torus compact neighborhood of* K *and* $N = \mathrm{Cl}(S^3 \smallsetminus T)$.