Midterm 1, Part 2, April 8, 28 points in 5 questions

Name and the student number:

Q1. (6 pts) *Show that map* $f\colon \mathbb{R}\mathrm{P}^1 \to \mathbb{R}\mathrm{P}^2$, $f([x_0 : x_1]) = [x_0^2 : x_1^2 : x_0x_1]$ *is (1) well-defined, (2) smooth, (3) is an immersion, (4) is an embedding.*

Q2. (6 pts) *Consider map* $g\colon \mathrm{SO}_2 \times \mathbb{R}^2 \to \mathbb{R}^2$, $g(A, v) = Av$, *where* $A \in \mathrm{SO}_2$ *is an orthogonal* 2×2*-matrix and* $v \in \mathbb{R}^2$ *is a 2-vector.*

(1) *show that* 0 *is a critical value;*
(2) *show that for all* $v \in \mathbb{R}^2$, $v \neq 0$, v *is a regular value;*
(3) *find* $g^{-1}(1, 0)$.

Q3. (6 pts) *Consider smooth maps between manifolds* $f: X \to Y$ *and* $g: Y \to Z$.

(1) *Show that* $\deg(g \circ f) = \deg f \times \deg g$ *whenever all the degrees are well-defined.*
(2) *In the case of* $\dim X = \dim Z = n$ *and* $\dim Y = m < n$ *show that* $\deg(g \circ f) = 0$.
(3) *Show that any map* $f: X \to \mathbb{R}^n$, $\dim X = n$, *has degree* 0.

Q4. (4 pts) *Assume that* X *and* Y *are oriented manifolds,* $\dim X = n$, $\dim Y = m$, $\partial X \neq \varnothing$, $\partial Y = \varnothing$.

(1) *Determine if the product orientation of* $X \times Y$ *induces on its boundary* $\partial(X \times Y)$ *the same orientation as the product orientation on* $\partial X \times Y$, *where* ∂X *is endowed with the induced boundary orientation.*

(2) *Can we say similarly that the two orientations on* $\partial(Y \times X) = Y \times \partial X$ *coincide?*

Q5. (6 pts) *Consider maps* $f, g\colon T^2 \to T^2$, $f(z_1, z_2) = (z^2, z^3)$, *and* $g(z_1, z_2) = (z_1 z_2, z_1^2 z_2^{-1})$, *where* $T \subset \mathbb{C}$ *is a unit circle.*

(1) *Determine if* f *and* g *are it orientation preserving ?*
(2) *Find the degrees of* f *and* g.