Midterm 1, Part 1

April 8, 50 points in 4 questions

Name and the student number:

- Q1. (16 pts) Explain briefly the following notions, and state the corresponding claims. (1) What is a critical point of a map $f: M \to N$? What is a critical value of f?
 - (2) What is an atlas of class C^r on a manifold ?
 - (3) What is a differential structure of class C^r on a manifold ? How it can be defined?
 - (4) State Sard's theorem.
 - (5) What is a smooth isotopy of diffeomorphisms?
 - (6) What is a Lie group ?
 - (7) What does mean "homogeneity of manifolds" (state the property)?
 - (8) What is the induced orientation on the boundary of an orientable manifold ?

Q2. (2 pts) Under which conditions on manifolds X, Y and Z the degrees deg f, deg g, deg $g \circ f$ for smooth maps $f: X \to Y$ and $g: Y \to Z$ are well-defined as integers ?

Q3. (8 pts) For the map $h: \mathbb{R}^2 \to \mathbb{R}P^2$, $h(x, y) = [x : y^2 : x^2y]$, find the critical points and the critical values.

Q4. (24 pts)

- (1) Show that chart $f : \mathbb{R} \to \mathbb{R}$, f(x) = 2x, defines the standard differential structure, C_{st} , in \mathbb{R} .
- (2) Show that chart $g: \mathbb{R} \to \mathbb{R}, g(x) = \begin{cases} x, & \text{if } x \le 0 \\ 2x, & \text{if } x \ge 0 \end{cases}$ defines a non-standard differential structure. C_{∞} in \mathbb{R} .
- ential structure, C_g , in \mathbb{R} . (3) Check if $h: \mathbb{R} \to \mathbb{R}$, $h(x) = x^3$ is a differentiable function with respect to the differential structure C_g . Find its class C^r .
- (4) Check if f and g are differentiable with respect to the smooth structure C_h defined by chart h.