Midterm 2, Part 1(take-home), due to May 14, 35 points in 6 questions
Name and the student number:

Q1. (9 pts) *Consider 2-dimensional vector bundle over* $\mathbb{R}\mathrm{P}^1$ *with the two charts* $\phi_i\colon U_i \to \mathbb{R}$, $i = 0, 1$, $U_i = \{[x_0 : x_1] \mid x_i \neq 0\}$, $\phi_0^{-1}(y) = [1 : y]$, $\phi_1^{-1}(x) = [x : 1]$.

(a) Verify that the coordinate change between these two charts can be described by $\phi_{01}(x) = \begin{pmatrix} x+2 & 1 \\ 2x & x \end{pmatrix}$ *and cannot be described by* $\phi'_{01}(x) = \begin{pmatrix} x+1 & 1 \\ 2x & x \end{pmatrix}$

(b) Write explicitly the relation between the basic vector fields $\{e_1, e_2\}$ *of* E *in chart* (U_0, ϕ_0) *and basic vector fields* $\{f_1, f_2\}$ *in chart* (U_1, ϕ_1).

(c) For a given vector field $V = 2xe_1 + (x^3 + 1)e_2$, *find its decomposition in* f_1 *and* f_2. *Can* V *be extended to the whole projective line* $\mathbb{R}\mathrm{P}^1$ *?*

Q2. (5 pts) *Consider vector field* $V = (x+2)\partial x$ *and 1-form* $\alpha = (x+2)dx$ *in the chart* $\{U_0, \phi_0\}$ *of* $\mathbb{R}\mathrm{P}^1$.

Calculate V *and* α *in the coordinate* y *of the chart* $\{U_1, \phi_1\}$ *and determine if* V *takes finite values at every point of* $\mathbb{R}\mathrm{P}^1$.

Q3. (3 pts) *Find the indices of the zeros of vector field* $V = (x^2 - 2y)\partial x + (y^2 - 2x)\partial y$ *in* $\mathbb{R}^2$.

Q4. (6 pts) *Consider mapping* $f\colon \mathbb{R}\mathrm{P}^2 \to \mathbb{R}\mathrm{P}^2$, $[x : y : z] \mapsto [x^2 : y^2 : z^2]$ *and a line* $L = \{Ax + By + Cz = 0\} \subset \mathbb{R}\mathrm{P}^2$.

(a) Under which conditions on A, B *and* C *this line is transversal to* f *?*

(b) Find the equation of L *and* $f^{-1}(L)$ *in the chart of* $\mathbb{R}\mathrm{P}^2$ *with the domain* $z \neq 0$.

Q5. (6 pts) *Consider smooth maps between manifolds* $f\colon X \to Y$ *and* $g\colon Y \to Z$*, and a submanifold* $L \subset Z$*.*

Prove that $g \circ f$ *is transverse to* L *if and only if the two conditions are satisfied:*
(1) g *is transverse to* L *and (2)* f *is transverse to* $g^{-1}(L)$*.*

Q6. (6 pts) *Give an example of a partition of unity on* $\mathbb{RP}^1$ *subordinate to the open covering by charts* U_1 *and* U_1*.*