

Midterm 2, Part 2

May 14, 32 points in 4 questions

Name and the student number:

Q1. (10 pts)

- (1) *State the local triviality condition for a vector bundle.*

- (2) *What is an atlas of vector bundle ? What are the coordinate change maps for vector bundle charts ?*

- (3) *What does mean “a smooth vector bundle” ?*

- (4) *What is an orientation of a vector bundle ? What does mean “non-orientable vector bundle” ?*

- (5) *What is a Riemannian metric in a vector bundle E ? How it can be interpreted as a tensor field ?*

- (6) *How a Riemannian metric allows to identify a vector bundle with its dual ?*

- (7) *What is a partition of unity subordinate to a given open covering $\{U_\alpha\}$?*

Q2. (8 pts) (1) *What does mean “map f is transversal to a submanifold” ?*

(2) *What does mean “submanifolds X and Y are transversal in Z ” ?*

(3) *What is the dimension of a transverse intersection of two n -dimensional submanifolds inside m -dimensional manifold ? For which n and m this intersection must be empty ?*

(4) *State the theorem about the preimage of a submanifold with respect to a map transversal to that submanifold.*

(5) *What is the intersection index of submanifolds and in which setting is it defined ? What is the self-intersection index of a submanifold ?*

(6) *What does mean “stability” ? Give example of stable properties.*

Q3. (8 pts) (1) *What is an isolated zero of a vector field and how to find its index ?
What does mean its non-degeneracy ?*

(2) *Give an example of a vector field and its zero, whose index is different from ± 1 .
Explain why.*

(3) *What is a degenerate critical point ? What is a Morse function ?*

(4) *What is the Milnor number of a critical point ?*

Q4. (6 pts) Determine if the map $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$, $f(x, y) = (x + y^2, xy)$, is transverse to the line $\{x + y = 1\}$.