# Knots, Links, and Braids

Week 1.

1. Knots and Links (oriented or not) in R3 and S3; diagrams; wild and tame knots; isotopy and homeomorphism; obverse links and reverse knots (for oriented ones).

2. Sum of knots, prime knots, the crossing number, the table of knots.

3. Reidermeister moves, regular isotopy of diagrams, Reidermeister theorem.

4. Braids with n strings, braid group, generators and relations, closed braids; Alexander theorem. Examples: (p,q)-torus knots (links). Markov moves and Markov theorem.

Task: show achirality of the figure-8 knot; find a braid presentation of (m,n)-torus link.

Week 2.

1. Spanning surface, calculation of its Euler characteristics, classification of surfaces.

2. Seifert surface: existence (Pontryagin, Frankl, Seifert), Seifert circuits. Genus of a knot g(K), its additivity, characterization of an unknot. Examples: genus for braids, torus knots.

3. Linking number, meridian and longitude. Writhe.

Week 3.

1. Satellite knots. Whitehead double, (p,q)-cable knots.

2. Alexander polynomial: the crossing-region and the crossing-arc methods of calculation; the invariant meaning, properties.

3. Seifert form and Alexander polynomial. Discriminant and signature of a knot.

Week 4.

1. Skein relation, the Convey polynomial. Calculation for tefoil knots. The unknotting number u(K).

2. Algebraic links. Milnor fibers, open book decomposition. Examples: torus knots, Briscorn spheres.

3. Fibrations, mapping tori, monodromy. Relation between the monodromy and Seifert form. Alexander polynomial as the characteristic polynomial.

Week 5.

1. 3-manifolds as open books. All orientation preserving homeomorphisms of s spere are homeomorphic. Dehn twist.

2. Mapping class group: generators and relations. Action in homology. Case of T2

3. Branch coverings, Rieman-Hurwits formula.

Week 6.

1. The monodromy of a branch covering. Factorization in the case of a disk. The boundary monodromy.

2. The effect of change of the basepoint. Hurwitz moves.

3. Example of a Hurwitz factorization: s1=(123),s2=(14),s3=(12)(34). Ramification profile at infinity. Case of double coverings: evenness of the number of branch points.

Week 7.

1. Term: ramification profile of a point. Hurwitz factorization of polynomials. Braid factorization: example with a cubic polynomial z3-z.

2. Braid group as π1(Cn-Δ).

3. Models of surface double branched coverings (with and without boundary). Mapping class groups of surfaces with marked points, the case of non-empty boundary. Branched coverings of higher-dimensional manifolds, local models.

Week 8.

1. Boundary Dehn twist via Garside element. Monodromy of open books via double covers branched along braids. Existence of open book decompositions of 3-manifolds.

2. Heegaard decomposition of 3-manifolds: solid handlebodies and their gluing.

Week 9.

1. Morse functions, non-degenerate critical points, Morse Lemma, gluing p-cells for index p critical pt.

2. Morse modifications: handles of index p and Morse surgery. Examples: surfaces and 3-manifolds.

3. Heegaard diagrams. Examples: the Lens spaces L(p,q).

Week 10.

1. Braid monodromy of curves in C2 (CP2).

2. Simple singularities and their braid monodromy.

3. Existence of double branched coverings in higher dimensions.

Week 11.

1. Cobordisms: oriented and non-oriented, the groups in the Low dimensions.

3. Dehn surgery.

Week 12.

1. Kirby moves. Slam-dunk. Examples: (p/q)-framed unknots, (p,q)-framed Hopf link, E\_8.

2. Heegaard diagram of the result of a surgery along (p/q)-framed unknot.

3. Disc bundles over a sphere. Self-intersection index. Plumbing 4-manifolds.

Week 13.

1. Morse functions on 4-manifolds, 1-handles and 2-handles, Mazur manifold, Akbulut’s cork.

2. Kirby diagrams presenting a) compact 4-mfds, 3-mfds, and closed 4-mfds.

3. Kirby Calculus: Akbulut’s dotted circle, Cancelation of handles, the Lattices of 4-manifolds (definite/indefinite, odd/even), examples of Lattices and classification.

Week 14.

1. Complex surfaces and symplectic manifolds: examples. Almost complex manifolds: b2++b1 is odd, 3σ+3Ϭ=c12

2. Blowup. Algebraic surfaces: Rational, elliptic, general type. Examples: hypersurfaces and double planes. Calculation of b2+ and b2-, the Euler characteristic and signature of double coverings.

3. Lefschetz pencils and fibrations. Elliptic fibrations. K3

Week 15.

1. Donaldson: existence of Lefschetz pencils compatible with a simplectic structure (definition of compatibility and Gompf’s theorem). Versions of Lefschetz fibrations (PALFs).

Monodromy factorization. Examples: E(n). Calculation of the Euler characteristic and the number of singular fibers in E(n). Abelinization of Map(F) for g=1,2.

Planned but missing topics

1. Lefschetz fibrations with boundary. Stein filling of open books.

2. Contact manifold, examples (contact space and contact lifting, links of singularities).

Legendrian and transverse curves. Tb-invariant and rotation number. Taut and overtisted. Symplectic filling.

3. Legendrian surgery and Stein fillings.

Planned:

Exotic structures: 7-spheres, h-cobordism. R4, Smoth Poincare conjecture, CP2, CP1 in CP2. Donaldson Theorem. Seiberg-Witten Invariants.