Midterm 2, Part 2

December 9 2013, 17:40 - 19:00, 32 points in 4 problems

Name and the student number:

Q1. (8 pts) (a) Given a 6-fold branched covering $S^2 \to S^2$ with 2 branch points of ramification index 3 and n simple branch points, find n.

(b) Find the genus g and the number of boundary components n on a surface $F_{g,n}$ which is a double cover of a punctured torus $F_{1,1}$ branched at 5 points.

Q3. (4 pts) Let $F \to D^2$ be a double covering branched at 3 vertices of a triangle ABC. Sketch F with the preimages of the sides AB, BC, and AC.

Q4. (16 pts) Let $f: F \to D^2$ be a 4-fold covering branched at 2 points, with the Hurwitz monodromy factorization $\mu_1, \mu_2 \in S_4$, where $\mu_1 = (123)$ and $\mu_2 = (34)$. (a) Find the monodromy along the boundary. How many connected components are in ∂F ?

(b) Show that another branched covering $f': F' \to D^2$ with the monodromy factorization $\mu_1 = (234)$ and $\mu_2 = (14)$ is isomorphic to the initial one. Explain what kind of this isomorphism using a commutative diagram of maps.

(c) Performing a Hurwitz move to μ_1 , μ_2 , give a new monodromy factorization μ_1'' , μ_2'' .

(d) Find some $\mu_1, \mu_2 \in S_4$ such that F is connected, but ∂F is not. Explain your answer.