MIDTERM II (TAKE-HOME)

Problem 1. *Let $f\colon T^2 \to T^2$, $F = t_a^2 t_b t_a^{-1}$, where a and b are the standard meridian and longitude on a torus.*

(1) *Determine the matrix of the induced map $f_*\colon H_1(T^2) \to H_1(T^2)$.*
(2) *Show that f is a product of right Dehn twists. Sketch the curves defining these Dehn twists.*
(3) *Determine the homology groups of the mapping tori M_f and M_{f^2}.*

Problem 2. *Let X_c denote the 3-manifold obtained from the standard Heegaard diagram of S^3 of genus 1 after changing the gluing map of the tori by a Dehn twist $t_c\colon T^2 \to T^2$. Sketch a Kirby diagram of X_c for*

(1) $c = a$
(2) $c = a + b$
(3) $c = 2a + b$

where a, b are the meridian and longitude of the torus T^2.

Problem 3. *Let $X \to \mathbb{C}\mathrm{P}^2$ be a double covering branched along a non-singular curves $A \subset \mathbb{C}\mathrm{P}^2$ of degree $2k$.*

(1) *find $b_2^{\pm}(X)$, $\chi(X)$ and $\sigma(X)$;*
(2) *determine $c_1(X)$ (as a multiple of the "hyperplane class" $h \subset H^2(X)$ coming from $\mathbb{C}\mathrm{P}^2$, and find $c_1^2(X)$.*
(3) *determine the Hodge diamond of X;*
(4) *for which k the surface X is rational ? For which k it is* Spin *?*

Problem 4. *Prove that a symplectic simply connected 4-manifold X must have $b_2^+(X)$ odd.*