PROBLEM LIST V

Alexander Polynomial and the Torsion

Problem 1. *Consider a real vector space V as a Λ-module, $\Lambda = \mathbb{R}[t, t^{-1}]$, where t act on V as an invertible linear operator $T\colon V \to V$. Show that*

(1) *V is a torsion module (that is contains no free summands over Λ);*
(2) *the torsion $|V|_\Lambda$ coincides with the characteristic polynomial, Δ_L, of L.*

Problem 2.

(1) *Find a Seifert matrix, S, and the Alexander polynomial Δ_K of a torus $(2, 2n+1)$-knot K.*
(2) *Compare your result with the characteristic polynomial of the monodromy matrix $H = S^T S^{-1}$ for this knot.*
(3) *Represent H as a product of the Dehn twists around the generators of the Seifert surface.*

Problem 3. *Consider the mapping torus M_f of a homeomorphism $f\colon T^2 \to T^2$, where $f = t_a t_b$ is a product of Dehn twists around the generators a, b of the torus T^2.*

(1) *Describe generators of $C_* = C_*(\widetilde{M}_f)$ over $\Lambda = \mathbb{Z}[t, t^{-1}]$ (specifying the corresponding cells in $\widetilde{M}_f$).*
(2) *Describe the differentials in the complex C_*.*
(3) *Describe the complex $C_*(M_f; \phi)$, where $\phi\colon \Lambda \to \mathbb{C}$ is a representation mapping t into $z \in \mathbb{C}^*$.*
(4) *Determine for which values of z the homology $H_*(M_f, \phi)$ vanish.*
(5) *Determine the Reidemeister torsion for those values z.*
(6) *Observe the relation between the Reidemeister torsion and the Alexander polynomial in this example.*