METU Department of Mathematics Math 748 Symplectic Topology Fall 2014 S.Finashin

Final, Theoretical questions

January 5, 84 points in 5 questions

Name and the student number:

Q1. (20 pts) *(a) What does mean a "pencil of algebraic curves" ?*

(b) What does mean a "J-holomorphic" or 'pseudo-holomorphic" curve ?

(c) What is the signature of a manifold ? For which manifolds we can define it ?

(d) What is a K3-surface ?

(e) What is a Fubini-Studi metric ? Where it is defined ? How does it show that complex algebraic manifold are symplectic ?

Q2. (16 pts) *(a) What is a Lefschetz fibration ?*

(b) What does mean: "a symplectic structure is compatible with a Lefschetz fibration" ?

(c) State Gompf's and Donaldson's theorem relating symplectic structures and Lefschetz fibration.

(d) What is meant by "the monodromy of a Lefschetz fibration" ? (In the other words, explain the relation between the Lefschetz fibrations and the mapping class group).

Q3. (12 pts) *(a) Give an example of a complex manifold which does not admit a symplectic structure.*

(b) Why your example is a complex manifold ?

(c) Why it does not admit a symplectic structure ?

Q4. (20 pts) *(a) Give an example of a differential form of type* $(2,1)$ *in* $\mathbb{C}^2$.

(b) Write a formula of the Hodge decomposition of cohomology into the sum of subgroups $H^{p,q}$. *What are the Hodge numbers* $h^{p,q}$ *?*

(c) Which kind of manifolds admit a Hodge decomposition as above ?

(d) What is the "Hodge diamond" ? What is it in the case of $\mathbb{C}\mathrm{P}^2$ *?*

(e) What is the geometric genus of a complex surface ? State the Hodge index theorem.

Q5. (16 pts) *(a) Formulate a criterion of existence of an almost complex structure for a 4-dimensional manifold.*

(b) What is the signature of a 4-dimensional manifold ?

(c) How the signature of a 4-manifold is related to its Pontryagin numbers ?

(d) How the Euler characteristics of complex surfaces is related to the Chern numbers ?