Test 2

(Take-home)

Q1. (2 pts) *Justify that* $Sp(\mathbb{R}^2) = SL_2(\mathbb{R})$.

Q2. (4 pts) *Find* W^ω *for the following subspaces in* $\mathbb{R}^4$ *with respect to the canonical symplectic form* $\omega = e_1^* \wedge f_1 + e_2^* \wedge f_2$.

(1) $W = \operatorname{span}(e_1 + e_2)$;
(2) $W = \operatorname{span}(e_1, f_2)$;
(3) $W = \operatorname{span}(e_1 + e_2, f_2)$.

Are there isotropic, symplectic and Lagrangian subspaces among these examples ?

Q3. (5 pts) *Consider vector field* $V = (x+1)\frac{\partial}{\partial x} + y^2 \frac{\partial}{\partial y}$, *1-form* $\alpha = (x+2y)dx - 3dy$, *and* $\omega = dx \wedge dy$.

(1) *Find* $f = \alpha(V)$;
(2) *find* $i_V(\omega)$;
(3) *find a vector field* W *such that* $i_W(\omega) = \alpha$;
(4) *find* V, α *and* ω *in the polar coordinates.*