## METU Department of Mathematics Fall 2014

**Math748 Symplectic Topology**

Instructor: Sergey Finashin

**The main Reference Books:**

1. Ana Cannas da Silva “Lectures on Symplectic Geometry” http://www.math.ethz.ch/~acannas/Papers/lsg.pdf
2. Dusa McDuff and Dietmar Salamon “Introduction to Symplectic Topology”

**Syllabus**

1. Linear Symplectic Geometry; Linear Darboux Theorem; Lagrangians; Linear Symplectic group; volume element; relation to Almost Complex Structures; Sp(2n), O(2n), GL(n,C), and U(n);
2. Review of differential forms; isotopies and vector fields, Lie derivatives (Flow, Lie brackets) and de Rham cohomology, cup product and Poincare duality in de Rham cohomology;
3. First examples of symplectic manifolds and Lagrangian submanifolds; Symplectic form on the cotangent bundle, graphs of symplectomorphisms as Lagrangian submanifolds; Weinstein principle;
4. Symplectic and Hamiltonian vector fields, Flux, Cartan’s formula, Liouville theorem, Poisson bracket; Classical Mechanics;
5. Contact manifolds, first examples: contact manifold, symplectization and contactization; Legendrian and contact submanifolds; Reeb vector fields; Open books and Giroux correspondence Theorem
6. Legendrian curve diagrams, canonical framing Thurston-Bennequin invariant, twisting number; Legendrian surgery;
7. Isotopy and deformation equivalence; Moser’s theorem, Darboux’s theorem; Tubular neighborhoods, Weinstein’s neighborhood Theorem;
8. Symplectomorphisms, Arnold’s conjectures, The idea of Floer Homology
9. Almost complex structures, existence and contractibility; Riemann and Kaehler manifolds, Fubini-Studi metric; compatibility of metrics, almost complex structures and symplectic forms; integrability: Nijenhuis tensor, Newlander-Nirenberg Theorem;
10. Lefschetz fibrations, Gompf and Donaldson’s theorems; Kodaira embedding Theorem
11. Contact Back: convexity, symplectic fillings: weak and strong; Stein fillings and Milnor open books;
12. Chern classes via connections and via obstructions; Examples and applications for complex surfaces
13. Pseudoholomorphic curves; moduli space, stability and compactification;
14. Hodge theory, Kaehler decomposition, Kaehler potential; strictly plurisubharmonic functions; Fubini-Study form; Hodge diamond, Strong Lefschetz Theorem
15. Hodge operator, harmonic forms; Laplacian; self-duality and symplectic structures
16. “Symplectic camel theorem”, capacities; symplectic blowup

If time permits: Symplectic and Hamiltomian group action, symplectic reduction moment map

**Other References**

1. Denis Auroux Lecture Notes in Symplectic Geometry
2. Tabachnikov “Introduction to Symplectic topology” <https://www.math.psu.edu/tabachni/courses/symplectic.pdf>
3. Ionel Lecture Notes http://math.stanford.edu/~ionel/257A-a12.html
4. Yasha Eliashberg “Symplectic Topology in the nineties” Diff. Geom. Appl. 9 (1998) 59-88

http://www.sciencedirect.com/science/article/pii/S0926224598000187

1. Dusa McDuff “Symplectic Structures – new approach to Geometry”
2. V.I.Arnold “Symplectic Geometry and Topology”, 1997
3. V.I. Arnold “Mathematical Methods of Classical mechanics”
4. <http://www.isc.meiji.ac.jp/~takahiko/workshop/summer_school07/lecture_notes/note.pdf>
5. <http://www-irma.u-strasbg.fr/~maudin/Arnold.pdf>
6. <http://www.math.sunysb.edu/~dusa/ewmcambrevjn23.pdf>
7. <https://www.dpmms.cam.ac.uk/study/III/2005-06/SymplecticTopology/L1intro.pdf>
8. <http://www.maths.ed.ac.uk/~aar/papers/arnogive.pdf>
9. http://www.homepages.ucl.ac.uk/~ucahjde/sympcourse.htm

# Description of the course

**Difficulty and prerequisites:** this course has intermediate level of difficulty and is designed for the graduate students who have already studied Topology for at least one year, and obtained some basic knowledge of differential manifolds, differential forms (Calculus on Manifolds), and homology (Algebraic Topology). It is useful at least minimal acquaintance with vector bundles and characteristic numbers.

**Objectives of the course:** is to give some basic knowledge on symplectic manifolds and related knowledge of complex and contact manifolds, that is expected from a modern topologist, and which will allow to read original paper and understand research talks in various hot subjects (Mirror symmetry, J-holomorphic curves and Quantum homology, Floer Theory, etc.).

**The tasks and grading:** two midterm exams and the final one, percentage is 30%+30%+40%. In addition, there may be several homeworks with exercises that will be graded, which may give totally 15-20% of bonus points.

**Textbooks:** there is no single textbook which contains all necessary and sufficient information. I will try to follow the textbook of Ana Cannas da Silva at least in the first lectures and give references to other textbooks and lecture notes if needed.